



Coimisiún na Scrúduithe Stáit State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2022

APPLIED MATHEMATICS – HIGHER LEVEL

FRIDAY, 24 JUNE – AFTERNOON, 2:00 TO 4:30

Five questions to be answered. All questions carry equal marks.

A *Formulae and Tables* booklet may be obtained from the Superintendent.

Take the value of g to be 9.8 m s^{-2} .

Marks may be lost if necessary work is not clearly shown.

Marks may be lost for omission of correct units with numerical answers.

Diagrams are generally not drawn to scale.

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1. (a) A train takes 40 minutes to travel from rest at station A to rest at station B. The distance between the stations is 20 km. The train left station A at 10:00. At 10:15 the speed of the train was 32 km h^{-1} and at 10:30 the speed was 48 km h^{-1} .

The speed of 48 km h^{-1} was maintained until the brakes were applied, causing a uniform deceleration which brought the train to rest at B.

During the first and second 15-minute intervals the accelerations were constant.

- (i) Draw a speed-time graph of the motion.
- (ii) Find the time taken for the first 16 km.
- (iii) Find the deceleration of the train.

- (b) A ball E is thrown vertically upwards with a speed of 42 m s^{-1} .

T (< 8) seconds later another ball, F , is thrown vertically upwards from the same point with the same initial speed.

- (i) Find where ball E is after 5 s and the total distance it has travelled in this time.
- (ii) Prove that when E and F collide, they will each be travelling with speed $\frac{1}{2}gT$.

2. (a) A ship is travelling at 22 km h^{-1} in a direction west 30° north. A boat sets out to intercept the ship from a point 25 km south of the ship.

The speed of the boat is 55 km h^{-1} .

Find

- (i) the direction the boat should steer
- (ii) the time, to the nearest minute, that it takes the boat to intercept the ship
- (iii) the distance between the boat and the ship 10 minutes before they meet.

- (b) A woman can swim at $u \text{ m s}^{-1}$ in still water. In a river she can cover a distance $d \text{ m}$ against the current in time t_1 and the same distance with the current in time t_2 . The current flows parallel to the straight banks at $v \text{ m s}^{-1}$.

- (i) Show that $v = \frac{d(t_1 - t_2)}{2t_1 t_2}$.

The width of the river is $d \text{ m}$ and $v < u$.

- (ii) Find, in terms of t_1 and t_2 , the time taken by the woman to cross the river by the shortest path.

3. (a) A particle is projected out to sea from a point P on a cliff to hit a target 60 m horizontally from P and 60 m vertically below P .

The velocity of projection is $14\sqrt{3} \text{ m s}^{-1}$ at an angle α to the horizontal.

Find

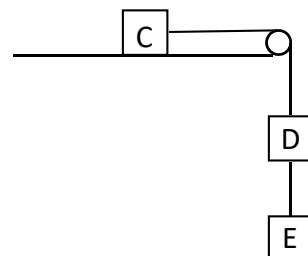
- (i) the two possible values of α
- (ii) the times of flight.

- (b) A particle is projected up a plane with speed $u \text{ m s}^{-1}$ at an angle β to the plane. The plane is inclined at 30° to the horizontal.

The plane of projection is vertical and contains the line of greatest slope.

Find the greatest range up the plane in terms of u .

4. (a) A block C of mass $6m$ rests on a rough horizontal table. It is connected by a light inextensible string which passes over a smooth fixed pulley at the edge of the table to a block D of mass $3m$. D is connected by another light inextensible string to a block E of mass $2m$, as shown in the diagram.

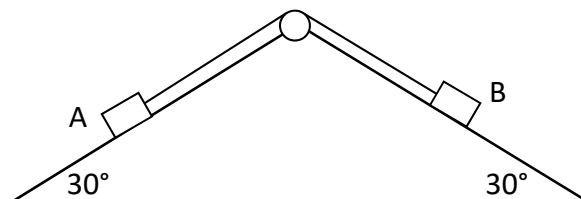


The coefficient of friction between C and the table is $\frac{1}{3}$.

The system is released from rest.

- (i) Show on separate diagrams the forces acting on each block.
- (ii) Find the acceleration of C.
- (iii) Find the tension in each string.

- (b) Particles A and B of masses m and $2m$ are connected by a light inextensible string which passes over a pulley at the top of a wedge, one particle resting on each of the faces, which are smooth.



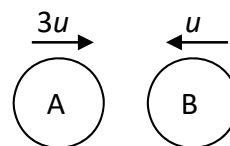
Each of the inclined faces of the wedge makes an angle of 30° with the horizontal.

The wedge of mass $3m$ rests on a smooth horizontal table.

The system is released from rest.

Find the acceleration of the wedge.

5. (a) A smooth sphere A of mass $2m$, moving with speed $3u$ on a smooth horizontal table collides directly with a smooth sphere B of mass m , moving in the opposite direction with speed u .



The coefficient of restitution between A and B is e .

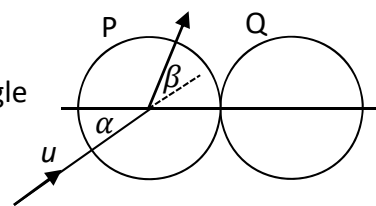
Find, in terms of u and e ,

- (i) the speed of each sphere after the collision
- (ii) the magnitude of the impulse imparted to B due to the collision.

The loss of the kinetic energy due to the collision is $km u^2(1 - e^2)$.

- (iii) Find the value of k .

- (b) A smooth sphere P has mass m and speed u . It collides obliquely with a smooth sphere Q, of mass m , which is at rest. Before the collision, the direction of P makes an angle α with the line of centres, as shown in the diagram.



The coefficient of restitution between the spheres is $\frac{1}{3}$.

During the impact the direction of motion of P is turned through an angle β .

Show that $\tan \beta = \frac{2 \tan \alpha}{1 + 3 \tan^2 \alpha}$.

6. (a) A particle moves on a straight line with simple harmonic motion about point O as centre. Its displacement from O at any time t is x .

At time $t = 0$ the particle passes through a point H at a distance of 3 cm from O , moving away from O . The particle next passes through H at time $t = 4$ s, moving towards O , and it passes through H for a third time after a further 12 s.

- (i) Find the period of the motion.
- (ii) Show that $x = A \sin(\omega t + \varepsilon)$, where A , ω and ε are constants, satisfies the differential equation

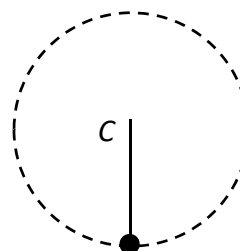
$$\frac{d^2x}{dt^2} = -\omega^2 x.$$

- (iii) Find the values of A , ω and ε for the particle.

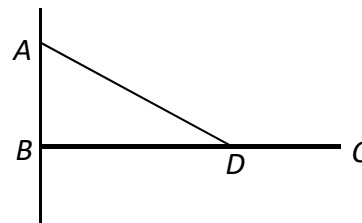
- (b) A particle is attached to one end of a light inextensible string of length 0.5 m. The other end of the string is attached to a fixed point C . The particle moves in a vertical circle.

The greatest and least tensions in the string are $3T$ and T , respectively.

Find the speed of the particle at the lowest point.



7. (a) A uniform rod BC of length 3 m, has a mass of 20 kg. The end B , about which the rod can turn freely, is attached to a vertical wall. The rod is kept in a horizontal position by a rope attached to a point D on the rod and to a point A of the wall vertically above B , as shown in the diagram.



$|AB| = h$ m and $|BD| = 2$ m.

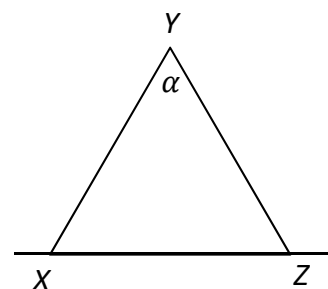
- (i) Prove that the tension in the rope is $\frac{147\sqrt{h^2+4}}{h}$.
- (ii) If the tension in the rope cannot exceed 245 N, show that $h \geq 1.5$.

- (b) Two uniform rods XY and YZ of equal length and of weights $2W$ and W respectively are smoothly hinged at Y .

The rods are at rest in a vertical plane with ends X and Z on a rough horizontal plane.

$|\angle XYZ| = \alpha$.

If the coefficient of friction is $\frac{\sqrt{3}}{5}$, find the maximum value of α such that the rods remain at rest.

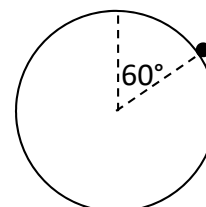


8. (a) Prove that the moment of inertia of a uniform disc, of mass m and radius r about an axis through its centre, perpendicular to its plane, is $\frac{1}{2}mr^2$.

- (b) A uniform disc of mass $4m$ and radius 20 cm is free to turn about a horizontal axis through its centre perpendicular to its plane.

A particle of mass m is attached to the edge of the disc.

Motion starts from the position in which the radius to the particle makes an angle of 60° with the upward vertical.



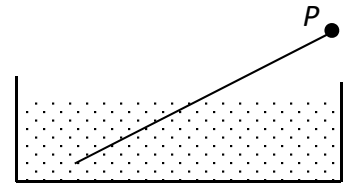
- (i) Find the angular velocity of the disc when the particle is at its lowest point.
- (ii) Find the angular displacement of the particle when the angular velocity of the disc is 5 rad s^{-1} for the first time.

9. (a) When placed in liquid A, a uniform solid cylinder floats upright with $\frac{2}{3}$ of its volume immersed in the liquid.

When placed in liquid B, the uniform solid cylinder floats upright with $\frac{4}{5}$ of its volume immersed in the liquid.

What fraction of the cylinder's volume is immersed when the cylinder floats upright in a uniform mixture of equal volumes of liquid A and liquid B?

- (b) A uniform rod, of length ℓ and weight W , is freely hinged at the point P .



The rod is free to move about a horizontal axis through P .
The other end of the rod is immersed in a liquid of density ρ .
The density of the rod is $s\rho$ ($s < 1$).

The rod is in equilibrium and is inclined as shown in the diagram.
The length of the immersed part of the rod is $x\ell$.

- (i) Find x in terms of s .
(ii) If the reaction at the hinge is $\frac{1}{6}W$ upwards, find the value of s .

10. (a) A particle moves in a horizontal line such that its speed v at time t is given by the differential equation

$$\frac{dv}{dt} = 5 - 8e^{-t}.$$

- (i) Given that $v = 2$ when $t = 0$, find an expression for v in terms of t .
(ii) Find the minimum value of v .
(iii) Find the distance travelled by the particle before it attains its minimum speed.

- (b) The rate of decay at any instant of a radioactive substance is proportional to the amount of the substance remaining at that instant. The initial amount of the radioactive substance is N and the amount remaining after time t (hours) is x .

- (i) Prove that $x = Ne^{-kt}$, where k is a constant.
(ii) If the initial amount N was reduced to $\frac{N}{3}$ in 14 hours, find the value of k .
(iii) If the amount remaining is reduced from $\frac{N}{3}$ to $\frac{N}{4}$ in t hours, find the value of t .

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Leaving Certificate Examination – Higher Level

Applied Mathematics

Friday, 24 June

Afternoon, 2:00 – 4:30