2026.M32

2026L020A1EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2026
Applied Mathematics

Higher Level

Tuesday 23 June Afternoon 2:00 - 4:30

400 marks

Examination Number	
Date of Birth	/ /
Centre Stamp	

For example, 3rd February 2005 is entered as 03 02 05

Instructions

There are ten questions on this paper. Each question carries 50 marks.

Answer any **eight** questions.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. All of your work should be presented in the answer areas, or on the given graphs, networks or other diagrams. Anything that you write outside of these areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet. If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You may lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in their simplest form, where relevant.

Diagrams are generally not drawn to scale.

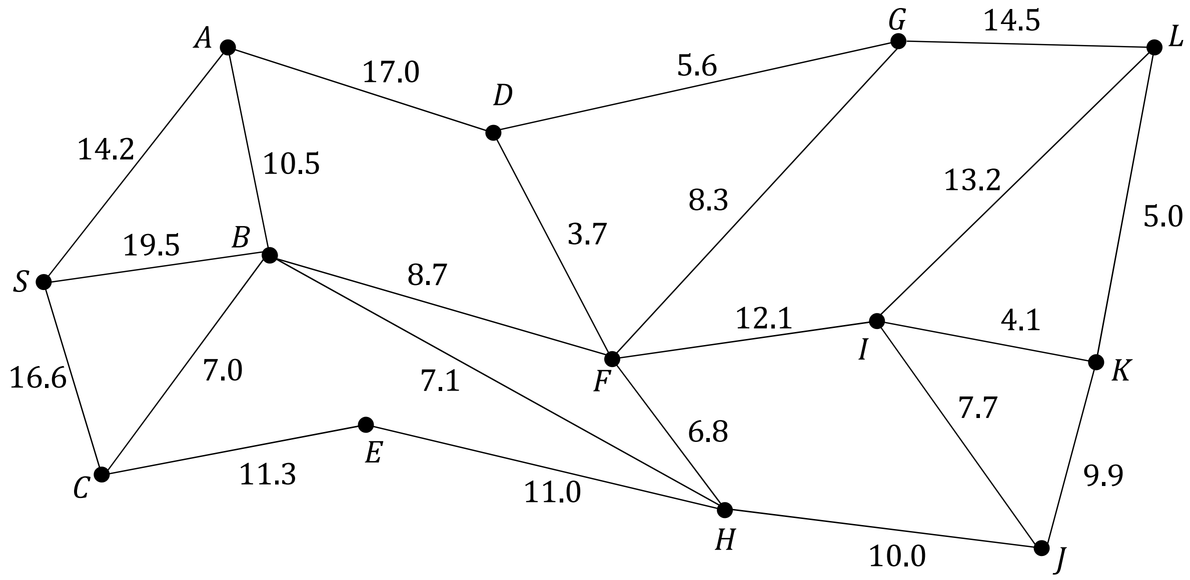
Unless otherwise indicated, take the value of g, the acceleration due to gravity, to be 9.8 m s^{-2}.

Unless otherwise indicated, $\vec{\imath}$ and $\vec{\jmath}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively, or eastwards and northwards, respectively, as appropriate to the question.

Write the make and model of your calculator(s) here:	

Question 1

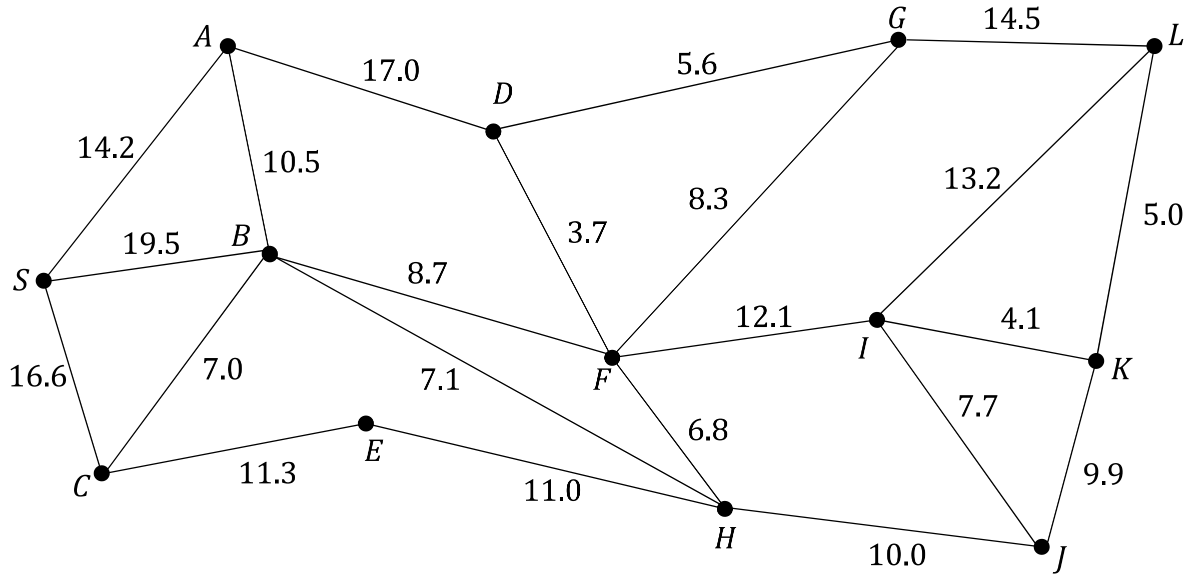
(a) A caretaker in a school checks that all the windows are locked in every classroom at the end of each school day.
In the network shown below the node labelled with the letter S represents the staffroom.
The nodes labelled with the letters A to L represent the locations of the classrooms. The weight of each edge represents the average time (in seconds) taken to walk between each classroom. The caretaker begins at the staffroom, S.



(i) Use an appropriate algorithm to find the minimum spanning tree for the network.
Name the algorithm you used. Relevant supporting work must be shown.

The caretaker finishes his check at classroom L. He then returns to staffroom S using the shortest path from L to S.

The network is redrawn below.



(ii) Use Dijkstra's algorithm to find the shortest path from L to S. Relevant supporting work must be shown.

(iii) On average, the caretaker takes 20 s to check each classroom. The caretaker checks every classroom by leaving the staffroom and using the minimum spanning tree. He then returns to the staffroom. Calculate the minimum total time taken.

(b) An entomologist wishes to model the change in a population of fruit flies. His model predicts the number of fruit flies (U_n), n days after a number of fruit flies are introduced to an environment where their population can increase.

Initially, five fruit flies are introduced to the environment, i.e. $U_0 = 5$.

He develops the first-order inhomogeneous difference equation below to model the change in population of fruit flies

$$U_{n+1} = 3U_n + 2n + 2$$

where $n \geq 0$, $n \in \mathbb{Z}$ and $U_0 = 5$.

(i) Calculate U_1 and U_2.

(ii) Show that $U_n = \frac{13}{2}(3^n) - n - \frac{3}{2}$ is a solution to the difference equation $U_{n+1} = 3U_n + 2n + 2$.

Question 2

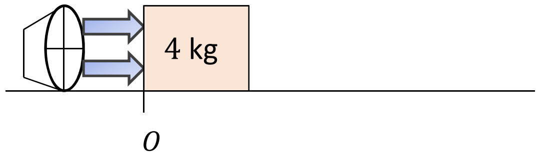
Karen is an electrical engineer who designs energy-efficient fans. There are seven activities (A to G) that need to be carried out to make a fan. The activities need to be carried out in a particular order.

Karen creates the table below by listing, for each activity, the other activities on which it depends directly. That is, for each activity $Y \in \{A, B, C, \dots, G\}$, she writes the smallest possible list of other activities which need to be completed before activity Y can begin.

Activity	**Depends directly on …**
A	—
B	—
C	A
D	A
E	B, C, D
F	B, D
G	B

(i) Draw a diagram of a directed graph to show the scheduling network for the assembly of a fan. The edges of your network should be drawn with non-dashed lines and labelled with the letters A to G. In addition, you may draw unlabelled edges (shown with dashed lines) to help explain the order in which the activities must happen.

Karen tests the efficiency of a fan by placing a 4 kg mass near the fan at point O on a rough horizontal floor. The coefficient of friction between the mass and the floor is $\frac{1}{5}$.



The mass is at rest before Karen switches on the fan.

When the fan is switched on, the mass is at O and the force exerted on the mass is 75 N. As the mass moves a displacement of x metres away from O, the force decreases by 12 N per metre.

(ii) Draw a diagram showing all the forces acting on the mass when it is in motion.

(iii) Show that the motion of the mass may be modelled by the differential equation

$$v\frac{dv}{dx} = 16.79 - 3x$$

where v is the velocity of the mass when its displacement is x.

(iv) Calculate v in terms of x.

(v) Calculate the maximum velocity of the mass.

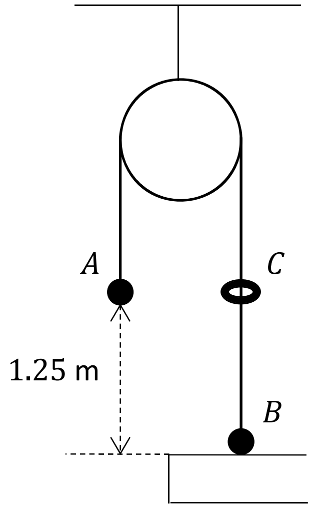
Question 3

Two particles A and B of mass $4m$ and m respectively are connected by a light inextensible string that passes over a smooth fixed pulley.

Particle A is held at a height of 1.25 m above the edge of a smooth horizontal table while particle B is at rest on the table, as shown in the diagram.

The string connecting particle A to particle B passes through a ring, C, of mass $8m$. C is held at the same height as particle A and directly above particle B.

At time $t = 0$ s particle A and particle B are released from rest.
C continues to be held at rest.



(i) Draw separate diagrams to show the forces acting on particle A and particle B when they are in motion and before particle B collides with C.

(ii) Calculate the acceleration of particle A and particle B before particle B collides with C.

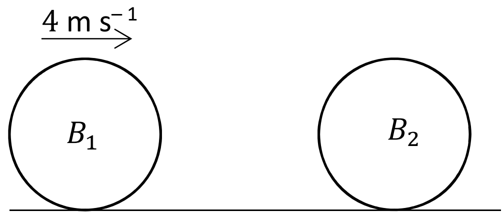
(iii) Calculate the velocity of particle A after it has fallen through 1.25 m.

Particle B collides with and sticks to C.

(iv) Calculate the velocity of the system immediately after particle B collides with C.

(v) Calculate the time taken for particle A to pass its original position after particle B collides with and sticks to C.

Question 4



Zhao strikes snooker ball B_1 to give it a velocity of 4 m s^{-1}. B_1 collides directly with a second snooker ball, B_2, which is at rest. Both balls have mass m kg.

The coefficient of restitution between the balls is e.

(i) Calculate, in terms of e, the velocity of B_1 and the velocity of B_2 after the collision.

The impulse imparted to B_2 as a result of the collision is $\frac{18m}{7}$ kg m s^{-1}.

(ii) Show that $e = \frac{2}{7}$.

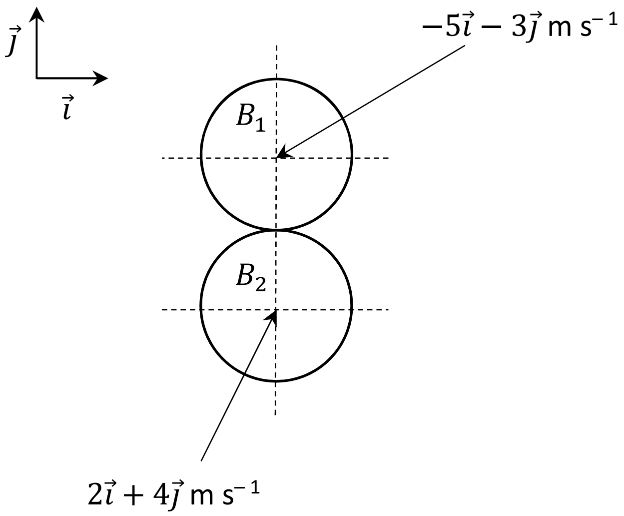
Later, Zhao strikes B_1 so that it collides obliquely with B_2.

Before the collision, B_1 has a velocity of $-5\vec{\imath} - 3\vec{\jmath}$ m s^{-1} and B_2 has a velocity of $2\vec{\imath} + 4\vec{\jmath}$ m s^{-1}.

The line joining their centres at the point of impact lies along the $\vec{\jmath}$ axis.

The coefficient of restitution, e, between the balls is $\frac{2}{7}$.

(iii) Calculate the speed of B_1 and the speed of B_2 after they collide.

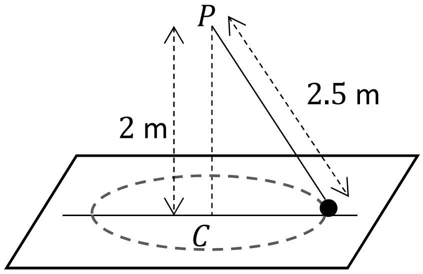


(iv) Calculate the angle through which B_1 is deflected as a result of the collision.

Question 5

A particle of mass of 5 kg is connected to point P by a taut light inextensible string of length 2.5 m. The particle has a constant speed v and describes a horizontal circle, of centre C, on a smooth horizontal table.

P is 2 m vertically above C.

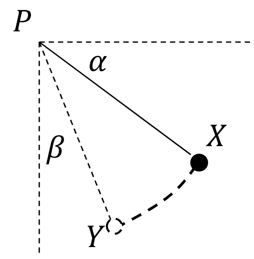


(i) Draw a diagram showing all the forces acting on the particle when it is in motion.

(ii) Calculate, in terms of v, the reaction force between the particle and the table.

(iii) Show that the particle will remain in contact with the table if $v \leq \frac{3\sqrt{2g}}{4}$.

The same particle is then held at a point X so that the taut string makes acute angle α with the horizontal. The particle is then released from X and moves through an arc of a vertical circle until it reaches point Y, where the string makes acute angle β with the vertical, as shown in the diagram.



(iv) Show that the speed of the particle at Y is

$$7\sqrt{\cos\beta - \sin\alpha}$$

(v) Calculate, in terms of α and β, the tension in the string when the particle is at Y.

Question 6

(a) The motion of a particle with velocity v and displacement s may be modelled by the differential equation

$$v\frac{dv}{ds} = 3v^2 + 4$$

The particle is released from rest, i.e. $v = 0$ when $s = 0$.

Solve this differential equation to find an expression for v in terms of s.

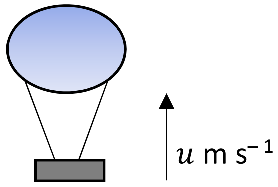
(b) A weather balloon leaves the ground at time $t = 0$.

The balloon rises vertically with constant upward velocity u m s^{-1}.

A sensor that is attached to the balloon is released when $t = t_1$.

The sensor lands on the ground t_2 seconds after the sensor is released.

Express t_1 in terms of t_2, u and g.



Question 7

A chef works in a kitchen with a constant background temperature of 20 °C.

The chef heats some caramel to 160 °C and then allows it to cool.

Using Newton's law of cooling, the chef develops a difference equation model to predict T_n, the temperature of the caramel after n minutes. She predicts that the temperature of the caramel may be calculated by

$$T_{n+1} = 20 + \lambda(T_n - 20)$$

where $n \geq 0, n \in \mathbb{Z}, \lambda \in \mathbb{R}$.

At $n = 0$ the temperature of the caramel is 160 °C, i.e. $T_0 = 160$.

At $n = 1$ the temperature of the caramel is 154.4 °C, i.e. $T_1 = 154.4$.

(i) By finding a value for λ, show that

$$T_{n+1} = 0.96T_n + 0.8$$

(ii) Solve this difference equation to find an expression for T_n in terms of n.

(iii) Calculate the minimum time, to the nearest minute, that the difference equation predicts that the caramel will take to cool below 80 °C.

The chef tries to improve the accuracy of her model by using a differential equation to represent the cooling of the caramel.

She assumes that the rate of change of cooling is proportional to the difference between the temperature of the caramel, T, and the background temperature of the kitchen.

She develops the differential equation

$$\frac{dT}{dt} = -\mu(T - 20)$$

where t is time in minutes and μ is a constant.

At $t = 0$ the temperature of the caramel is 160 °C.

At $t = 1$ the temperature of the caramel is 154.4 °C.

(iv) Solve the differential equation to find an expression for T in terms of t.

Question 8

(a) An object has an initial speed u m s^{-1}. It is given a uniform acceleration a m s^{-2} for n seconds.

s_n is the distance in metres travelled, in n seconds.

(i) Express s_n in terms of u, a and n.

D_n is defined as the distance travelled in the n^{th} second, where $n \in N$, i.e. $D_n = s_n - s_{n-1}$.

(ii) Express D_n in terms of u, a and n.

(iii) Derive a difference equation for D, i.e. express D_{n+1} in terms of D_n and a.

(b) Lily plants produce a number of bulbs every year.
Bulbs are planted so that they grow into new lily plants which may produce more bulbs the following year.

A horticulturist harvests the bulbs of the lily plants to sell some of them.

In 2023, she harvests 151 bulbs.

In 2024, she harvests 306 bulbs.

The horticulturist predicts that G_n, the number of bulbs harvested in year n, may be expressed by the second-order inhomogeneous difference equation

$$G_{n+2} = \frac{7G_{n+1} + 12G_n}{5} - 14n - 4$$

where $n \geq 0$, $n \in \mathbb{Z}$, $G_0 = 151$ and $G_1 = 306$.

(i) Solve the difference equation to find an expression for G_n in terms of n.

(ii) Calculate the number of bulbs that are predicted to be harvested in 2028.

Question 9

(a) A particle is initially held at point P. The particle begins to move under a force such that v, the velocity of the particle, may be modelled by the differential equation

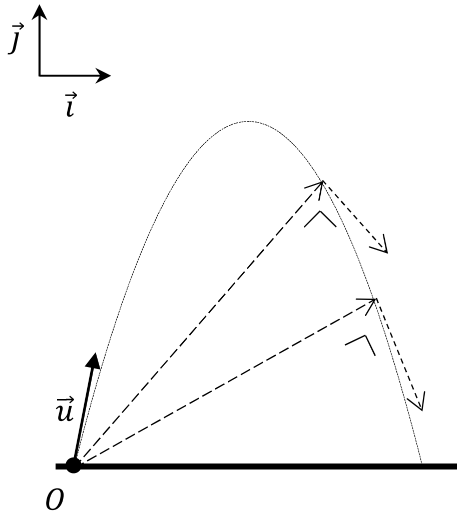
$$v = \frac{ds}{dt} = t^2 \ln 2t$$

where s is the displacement (in metres) of the particle from P at any time t (in seconds) and where $s = 0$ when $t = 0$.

(i) Use integration by parts to show that $s = \frac{t^3}{3}\left(\ln 2t - \frac{1}{3}\right)$.

(ii) Calculate the value for t when the particle will next reach P.

(b) An object is projected at time $t = 0$ seconds from point O on horizontal ground with an initial velocity $\vec{u} = 4.9\vec{\imath} + 14.7\vec{\jmath}$ m s^{-1}.



(i) Express $\vec{v}$, the velocity of the object at any time t, in terms of $\vec{\imath}$ and $\vec{\jmath}$.

(ii) Express $\vec{r}$, the displacement of the object at any time t, in terms of $\vec{\imath}$ and $\vec{\jmath}$.

At time t_1 and then again at time t_2 the velocity of the object is perpendicular to its displacement.

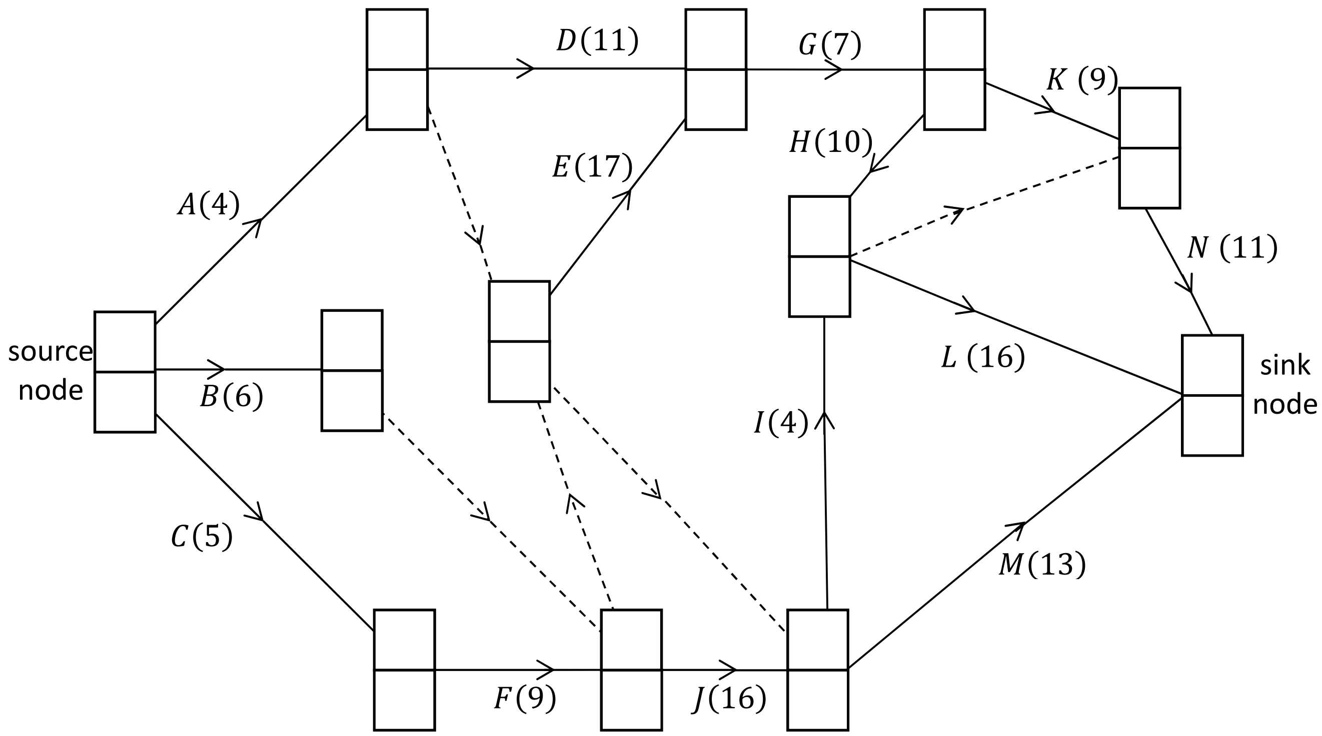
(iii) Show that $\frac{t_2}{t_1} = \frac{5}{4}$.

Question 10

The manager of a hotel is planning for their main function hall to be redecorated. The diagram below shows a directed graph of the scheduling network for the project.

The edges of the network represent the activities that have to be completed as part of the project and are labelled with the letters A to N. The duration, in hours, of each activity is represented by the number in brackets. The unlabelled edges (shown with dashed lines) do not represent real activities but they help explain the order in which the activities must happen. The letters used to label the edges should **not** be taken as representing the order in which the activities happen.

The nodes of the network represent events or points in time during the project. The source node is the time when the project begins and the sink node is the time when the project ends.



(i) Calculate the early time and the late time for each event.
Complete the diagram above by writing the early time (upper box) and late time (lower box) at the node representing each event.
Use the space below to show relevant supporting work, if necessary.

(ii) Write down the critical path(s) for the network.

A cascade chart (Gantt chart) is a type of bar chart which may be used to represent a project's schedule. The duration of each activity is represented by the width of the horizontal bar for that activity, with time on the horizontal axis. The float time for an activity is represented by a rectangle drawn using dotted lines to the right of the bar for that activity. The top row of a cascade chart is used for a critical path.

(iii) Draw a cascade chart or similar bar chart to represent the schedule for this project.

hours

4 8 12 16 20 24 28 32 36 40 44 48 52 56 60 64 68

The hotel manager must ensure that there is an adequate number of workers available to complete this project on time.

(iv) Assuming that there are no delays to any activity, calculate the minimum number of workers the hotel manager needs to employ so that this project will finish on time.
Justify your answer.

(v) If activity B was delayed by 7 hours, what effect would this have on the critical path(s) and on the time it takes to complete the project? Explain your answer.

Page for extra work.

Label any extra work clearly with the question number and part.

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Acknowledgements

Images

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