Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2026

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

General notes on the marking

In general, accept a candidate's work, whether correct or incorrect, for use in subsequent parts of the question, unless this oversimplifies the work involved.

Scales and Slips
Certain types of mistake – typically involving incorrect rounding, early rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work – are referred to as a **slip** and are denoted with a *. In such cases, a mark that is one mark below the full-credit mark may be awarded. This level of credit is referred to as *Full Credit –1*. Thus, for example, in Scale 10C, *Full Credit –1* of 9 marks may be awarded.

In general, an arithmetical error is where there are two numbers with an operator ($+$, $-$, $\times$, $\div$), and the answer is incorrect. As noted above, these are treated as slips.

The only marks that may be awarded for a question are those on the appropriate scales, or *Full Credit –1*.

Once a candidate has presented sufficient work to be awarded a particular level of credit, they are not awarded a lower level of credit, regardless of any subsequent errors in the work they present.

Instructions regarding penalties for omitted or incorrect units are given in the scheme for each question to which they apply. A penalty for rounding is applied once per unit of marking (i.e., if parts (a) and (b) are marked as a single unit, a penalty for rounding is only applied once for (a) and (b) combined).

In general, an answer without sufficient supporting work is awarded the lowest level of credit on the scale above *No Credit* in Paper 1 (typically *Partial Credit* or *Low Partial Credit*, as appropriate), and the highest level of credit on the scale below *Full Credit* in Paper 2 (typically *Partial Credit* or *High Partial Credit*).

Steps
Where steps are listed in the Marking Notes, unless otherwise specified, it is to be taken that they can be independently correct / incorrect – that is, in a candidate's solution, step n can be considered correct even if previous step(s) have not been correctly presented, as long as the work done to arrive at step n from the previous step(s) has not been oversimplified. Note also that these steps may not need to be presented in the order specified in the Marking Notes. In general, it is specifically noted where these do not hold.

Errors
An error is a mistake in a candidate's work that is not considered a slip. Where a question is **not** marked using steps, if a candidate has a single error, they are generally awarded one level of credit below that which they would otherwise have been awarded. Similarly, where they have two errors, they are generally awarded two levels of credit below that which they would otherwise have been awarded. (If they present sufficient work for *Low Partial Credit*, they will be awarded this at a minimum, regardless of the number of errors.) For example, on a C-scale:

- *High Partial Credit*: One error, otherwise fully correct (or fully correct with a *)
- *Low Partial Credit*: Two errors, otherwise fully correct (or fully correct with a *)

Where a question **is** marked using steps, this does not apply. Instead, an error in a step means that the step has not been completed correctly; this does not affect the completion of other steps (unless it oversimplifies the work). So, if a candidate has multiple errors on a single step, they could still be awarded up to *High Partial Credit*, depending on the marking scheme.

Slips and Errors
Where a candidate has a single * on their solution, this is ignored in the awarding of credit unless they would otherwise have *Full Credit*. If they would otherwise have *Full Credit*, then *Full Credit –1* is awarded. Where a candidate has multiple *s, this is generally treated as an error. This applies even where the *s are in different steps (though see note above regarding work sufficient for a particular level of credit).

Multiple answers
Where a solution requires substantial work, all separate attempts are marked and the marks for the best one are awarded, regardless of crossing out.

Where a solution requires selection from the question:

- If a candidate has only one answer, which they have then crossed out, the crossing-out is ignored
- If a candidate has both crossed-out answer(s) and answer(s) that are not crossed out, ignore the crossed-out answer(s) and only mark those that are not crossed out
- If a candidate has multiple answers that are **not** crossed out, award the lowest mark associated with these answers (generally, such a set of answers will end up being considered incorrect).

Excess work
Where a solution requires substantial work, incorrect excess work (i.e. work presented after the answer) is generally not penalised.

This does not apply where candidates are asked to write a sentence, or number of sentences, to explain / justify / describe something. In such cases, the answer as a whole is considered.

Where a subsequent part of a question requires candidates to use their answer from a previous part, and an incorrect answer arising from incorrect excess work is used, this is generally treated as an error in the subsequent part, unless it oversimplifies the work involved (in which case a lower level of credit is generally awarded for the subsequent part).

Model solutions
The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Square brackets
Where something is contained in square brackets in the model solution, it is **not** required for *Full Credit*.

Work of merit
Where the scheme indicates "work of merit" (or "substantial work of merit"), examples are given that exemplify the standard of work required to be considered work of merit (or substantial work of merit) in that particular question.

Accept
Where it is indicated to "accept" work, this means that, for this year, such work is to be accepted for *Full Credit*, unless otherwise indicated.

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✓	Tick	Correct step / part	The work presented in the body of the script merits full credit
✗	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
*	Star	Slip (Rounding / Unit / Arithmetic error / Misreading)	
	Horizontal wavy	Error	
P	P		The work presented in the body of the script merits *Partial Credit*
L	L		The work presented in the body of the script merits *Low Partial Credit*
M	M		The work presented in the body of the script merits *Mid Partial Credit*
H	H		The work presented in the body of the script merits *High Partial Credit*
F*	F star		The work presented in the body of the script merits *Full Credit* -1
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
	Vertical wavy	No work on this page / portion of this page	
O	Oversimplify	The candidate has oversimplified the work	
WOM	Work of merit	The candidate has produced work of merit (in line with that defined in the scheme)	
S	Stops early	The candidate has stopped early in this part	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work.

In a **C scale** that is **not** marked using steps, where * and and appear in the body of the work, then L should be placed in the right margin.

In the case of a **D scale** with the same annotations, M should be placed in the right margin.

Note: Where work is presented for a question in the answerspace provided, and a separate attempt is presented on additional pages, both attempts are marked.

For each attempt, the annotation for the appropriate credit level is put in the right margin next to that attempt. The annotation for the lower credit level is then left-bracketed, and the mark for the higher credit level is awarded.

Structure of the marking scheme – Paper 1

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled **A** divide candidate responses into two categories (correct and incorrect), scales labelled **B** divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	**B**	**C**	**D**
Number of categories	3	4	5
5-mark scales	0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5
10-mark scales	0, 5, 10	0, 3, 7, 10	0, 2, 5, 8, 10
15-mark scale			0, 3, 8, 12, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme.

Marking scales – level descriptors

B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

Model Solutions & Marking Notes – Paper 1

<table>
<tr><th>P1 Q1</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

$$5 = 1 + \sqrt{a+7}$$
$$\Leftrightarrow \quad 4 = \sqrt{a+7}$$
$$\Leftrightarrow \quad 16 = a + 7$$
$$\Leftrightarrow \quad a = 9$$

</td><td>

Scale 5C (0, 2, 3, 5)

Accept correct answer if verified.

Low Partial Credit
- Work of merit, for example, one relevant operation indicated or carried out

High Partial Credit
- One error, otherwise correct
- Linear equation in a

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<tr><td>(b)</td><td>

Method 1:

$\Leftrightarrow \quad \frac{2}{p} = \frac{3}{s} - \frac{1}{n}$ **...S1**

$\Leftrightarrow \quad \frac{2}{p} = \frac{3n-s}{sn}$

$\Leftrightarrow \quad \frac{p}{2} = \frac{sn}{3n-s}$ **...S2**

$\Leftrightarrow \quad p = \frac{2sn}{3n-s}$ **...S3**

Method 2:

$\Leftrightarrow \quad ps + 2ns = 3np$

$\Leftrightarrow \quad ps - 3np = -2ns$ **...S1**

$\Leftrightarrow \quad p(s - 3n) = -2ns$ **...S2**

$\Leftrightarrow \quad p = \frac{-2ns}{s-3n}$ **...S3**

Method 3:

$\Leftrightarrow \quad \frac{2}{p} = \frac{3}{s} - \frac{1}{n}$ **...S1**

$\Leftrightarrow \quad \frac{p}{2} = \frac{1}{\frac{3}{s} - \frac{1}{n}}$ **...S2**

$\Leftrightarrow \quad p = \frac{2}{\frac{3}{s} - \frac{1}{n}}$ **...S3**

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Scale 10D (0, 2, 5, 8, 10)

Note: a cross on its own (indicating cross-multiplication) is not considered work of merit.

Consider solution as consisting of 3 steps:

Step 1: Isolates term(s) with p
Step 2: Writes as $ap = b$, with a, b ind. of p
Step 3: Finds p

Low Partial Credit
- Work of merit, for example, finds a relevant common denominator

Mid Partial Credit
- 1 step correct

High Partial Credit
- 2 steps correct

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<tr><th>P1 Q1</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(c)</td><td>

1. Finding linear factor, Method 1:

$$\begin{array}{r} 4x-5 \\ 2x^2+4x-6\;\overline{\big)\,8x^3+6x^2-44x+30} \\ \underline{8x^3+16x^2-24x} \\ -10x^2-20x+30 \\ \underline{-10x^2-20x+30} \\ 0 \end{array}$$

Sets up long division **... S3**
Finds first term in quotient ($4x$) **... S4**

1. Finding linear factor, Method 2:

$$\begin{array}{c|c|c|c|} & 2x^2 & +4x & -6 \\ \hline 4x & 8x^3 & 16x^2 & -24x \\ \hline -5 & -10x^2 & -20x & 30 \\ \hline \end{array}$$

Sets up array **... S3**
Finds either $4x$ or -5 **... S4**

1. Finding linear factor, Method 3:

$(2x^2+4x-6)(ax+b)$ **... S3**
$= 8x^3+6x^2-44x+30$

x^3: $2ax^3 = 8x^3$ so $a = 4$

Constant: $-6b = 30$ so $b = -5$

Finds either $a = 4$ or $b = -5$ **... S4**

2. Finding roots:

$2x^2+4x-6 = 0$
$\Leftrightarrow \quad x^2+2x-3 = 0$
$\Leftrightarrow \quad (x+3)(x-1) = 0$ **... S1**
$\Leftrightarrow \quad x = -3$ or $x = 1$ **... S2**

And $4x-5 = 0$
$\Leftrightarrow \quad x = \frac{5}{4}$ **...S5**

Answer: $x = -3,\ x = 1,$ and $x = \frac{5}{4}$

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Scale 15D (0, 3, 8, 12, 15)

Accept answers if fully verified (including by substitution).

Consider **Methods 1 to 3** of solution as consisting of 5 steps:

Step 1: Factorises $2x^2+4x-6$ / fully substituted quadratic formula
Step 2: Finds two roots of quadratic factor
Step 3: Sets up long division / array / product
Step 4. Finds one term in quotient ($4x$ or -5)
Step 5. Finds $x = \frac{5}{4}$

Finds linear factor by observation, even if only one term is correct, is Steps 3 and 4 completed

Consider **Method 4** of solution (next page) as consisting of 5 steps:

Step 1: Finds 1 root
Step 2. Sets up long division / array / product
Step 3. Finds quotient
Step 4: Factorises quotient / fully substituted quadratic formula
Step 5: Finds two roots of quadratic factor

Low Partial Credit
- Work of merit, for example, trials (at least 2) values; 1 step correct
- Correct answers without supporting work

Mid Partial Credit
- 2 steps correct

High Partial Credit
- 4 steps correct

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P1 Q1	Model Solution – 30 Marks	Marking Notes
(c) cntd	**Method 4:** *Doesn't use given quadratic factor, for example:* $8(1^3) + 6(1^2) - 44(1) + 30 = 0$ So $x = 1$ is one root **...S1** and $x - 1$ is a factor Sets up division of $x - 1$ in to $8x^3 + 6x^2 - 44x + 30$ ... **...S2** ... to find $8x^2 + 14x - 30$ **...S3** Then $8x^2 + 14x - 30 = 0$ $\Leftrightarrow \quad (4x - 5)(2x + 6) = 0$ **... S4** $\Leftrightarrow \quad x = \frac{5}{4}$ or $x = -3$ **... S5**	*See previous page for marking notes.*

P1 Q2	Model Solution – 30 Marks	Marking Notes
(a)	$4{\cdot}\dot{3}\dot{6} \quad = 4 + \frac{36}{100} + \frac{36}{100^2} + \frac{36}{100^3} + \cdots$ This is $4 +$ a geometric series, with $a = \frac{36}{100}$ and $r = \frac{1}{100}$. Sum of geometric series: $\frac{\left(\frac{36}{100}\right)}{1-\frac{1}{100}}$ Then $4{\cdot}\dot{3}\dot{6} \quad = 4 + \frac{\left(\frac{36}{100}\right)}{1-\frac{1}{100}}$ $= 4 + \frac{36}{100-1}$ $= 4 + \frac{36}{99}$ $= \frac{432}{99}$ or $\frac{146}{33}$ or $\frac{48}{11}$	**Scale 10C (0, 3, 7, 10)** Accept an unsimplified or partially-simplified fraction, as long as it is in the form $\frac{a}{b}$ Award at most *Low Partial Credit* if an infinite geometric series is not used *Low Partial Credit* • Work of merit, for example, identifies a or r, **or** writes two relevant terms in geometric sequence *High Partial Credit* • Formula for sum of infinite geometric series substituted correctly *Notes re ** • Apply a * if the answer is in an incorrect form, for example, $4\frac{4}{11}$ • Apply a * if 4 [units] is not included

P1 Q2	Model Solution – 30 Marks	Marking Notes
(b) (i)	$(1+0{\cdot}084)^{\frac{1}{12}}-1=0{\cdot}00674\ldots$ $=0{\cdot}67\%$ [2 D.P.]	**Scale 5C (0, 2, 3, 5)** Note that considering $1{\cdot}0067^{12}$ and finding $8{\cdot}34\ldots\%$ is considered an error, so *High Partial Credit* at most Accept "$i=0{\cdot}00674\ldots=0{\cdot}0067$" *No Credit* • Correct answer ($0{\cdot}67\%$) without supporting work *Low Partial Credit* • Work of merit, for example, $0{\cdot}084$, $\frac{1}{12}$ or 0.67% converted to 0.0067 *High Partial Credit* • Indicates $(1+0{\cdot}084)^{\frac{1}{12}}$ or equivalent • Finds interest $=8{\cdot}3\ldots\%$ or $8{\cdot}4\%$ using $(1{\cdot}0067)^{12}=1{\cdot}0834\ldots$ *Note re ** • Apply a * if shows $(1+0{\cdot}084)^{\frac{1}{12}}-1=0{\cdot}0067$ but doesn't show rounding explicitly (i.e. doesn't show $0{\cdot}00674$)

P1 Q2	Model Solution – 30 Marks	Marking Notes
(b) (ii)	**Method 1**: $i = 0{\cdot}0067, P = 14\,000, t = 60$ Amortisation formula: $A \quad = P\,\frac{i(1+i)^t}{(1+i)^t-1}$ $= 14\,000\left[\frac{0{\cdot}0067(1{\cdot}0067)^{60}}{(1{\cdot}0067)^{60}-1}\right]$ **…S1** $= 284{\cdot}137\ldots$ $= [€]284{\cdot}14$ [2 D.P.] **…S2** So, interest $= 284{\cdot}14 \times 60 - 14\,000$ $= 17\,048{\cdot}40 - 14\,000$ $= [€]3048{\cdot}40$ **… S3** **Method 2**: [*Letting repayment amount be A, present value of loan is*:] $\frac{A}{1{\cdot}0067} + \frac{A}{1{\cdot}0067^2} + \cdots + \frac{A}{1{\cdot}0067^{60}} = 14\,000$ $\frac{1}{1{\cdot}0067} + \cdots + \frac{1}{1{\cdot}0067^{60}}$ $= \frac{\frac{1}{1{\cdot}0067}\left(1 - \frac{1}{1{\cdot}0067^{60}}\right)}{1 - \frac{1}{1{\cdot}0067}}$ **…S1** $\Leftrightarrow \quad A \times (49{\cdot}2719\ldots) = 14\,000$ $\Leftrightarrow \quad A = \frac{14\,000}{49{\cdot}2719\ldots}$ $= [€]284{\cdot}14$ [2 D.P.] **…S2** So, interest $= [€]3048{\cdot}40$ **…S3**	**Scale 15D (0, 3, 8, 12, 15)** Note: $t = 59$ is treated as an error. Accept $3048{\cdot}4$. Consider solution as consisting of 3 steps: **Step 1**: Fully correct substitution into geometric / amortisation formula **Step 2**: Finds value of A, based on geometric / amortisation formula **Step 3.** Finds interest, based on A from Step 2 *Low Partial Credit* • Work of merit, for example, writes $0{\cdot}67\%$ as a decimal, or $t = 60$ • Some correct substitution in geometric / amortisation formula *Mid Partial Credit* • 1 step correct *High Partial Credit* • 2 steps correct *Note re ** • Apply a * for incorrect rounding (including early rounding) • Apply a * if a more accurate value than $0{\cdot}67\%$ is used (for example, $0{\cdot}00674$).

P1 Q3	Model Solution – 30 Marks	Marking Notes
(a)	$u(x) = 7x^2$ so $u'(x) = 14x$ $v(x) = \cos(3x)$ so $v'(x) = -3\sin(3x)$ Then, $f'(x) = 7x^2(-3\sin(3x)) + \cos(3x)\,(14x)$ So $f'(\pi) = -21(\pi^2)\sin(3\pi) + \cos(3\pi)\,(14\pi)$ $= -14\pi$	**Scale 10D (0, 2, 5, 8, 10)** Accept correct $f'(x)$ without supporting work. Some correct differentiation required to get any credit. Calculator in the incorrect mode is treated as an error. *Low Partial Credit* • Work of merit in finding $f'(x)$, for example, finds $u'(x)$ *Mid Partial Credit* • Finds $f'(x)$ • Work of merit in finding $f'(x)$ **and** work of merit in finding $f'(\pi)$ (for example, some correct substitution of π in $f'(x)$) *High Partial Credit* • One part is correct (finds $f'(x)$ or $f'(\pi)$) **and** work of merit in the other part *Note re ** • Apply a * if the answer is in an incorrect form, for example, $-43{\cdot}98$ ...

P1 Q3	Model Solution – 30 Marks	Marking Notes
(b) (i)	$g'(x) = \frac{30x^2}{2} - 18x + k$ **… S1** $= 15x^2 - 18x + k$ $g'(-1) = 15(-1)^2 - 18(-1) + k$ **…S2** $g'(-1) = -2$ So, $15(-1)^2 - 18(-1) + k = -2$ **… S3** i.e. $k = -33 - 2 = -35$ **…F*** and $g'(x) = 15x^2 - 18x - 35$ **… S4**	**Scale 10D (0, 2, 5, 8, 10)** Note: differentiating the given function is not considered work of merit. Consider solution as consisting of 4 steps: **Step 1**: Integrates $30x$ and -18 **Step 2**: Subs $x = -1$ in expression for $g'(x)$ [which might not have a constant of integration] **Step 3**: Uses -2 correctly **Step 4.** Finds expression for $g'(x)$ Constant of integration is needed for Steps 3 or 4 to be correct *Low Partial Credit* • Work of merit, for example, indicates integration, or some correct integration (that is, a fully correct term), or substitutes in $x = -1$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Note re ** • Apply a * if all three terms in $g'(x)$ are found, but $g'(x)$ is not given as a single expression

P1 Q3	Model Solution – 30 Marks	Marking Notes
(b) (ii)	$g(x) \;= \frac{15x^3}{3} - \frac{18x^2}{2} - 35x + h$ **... S1** $= 5x^3 - 9x^2 - 35x + h$ $g(-1) = 5(-1)^3 - 9(-1)^2 - 35(-1) + h$ **...S2** $g(-1) = 8$, So, $5(-1)^3 - 9(-1)^2 - 35(-1) + h = 8$ **...S3** i.e. $h = 8 - 21 = -13$ **...F*** and $g(x) = 5x^3 - 9x^2 - 35x - 13$ **...S4**	**Scale 10D (0, 2, 5, 8, 10)** Consider solution as consisting of 4 steps: **Step 1**: Integrates all terms in $g'(x)$ from part **(b)(i)** **Step 2**: Subs $x = -1$ in expression for $g(x)$ [which might not have a constant of integration] **Step 3**: Uses 8 correctly **Step 4.** Finds expression for $g(x)$ Constant of integration is needed for Steps 3 or 4 to be correct *Low Partial Credit* • Work of merit, for example, some correct integration [that is, a fully correct term], or brings down $g'(x)$ from **(b)(i)** *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Note re ** • Apply a * if all four terms in $g(x)$ are found, but $g(x)$ is not given as a single expression

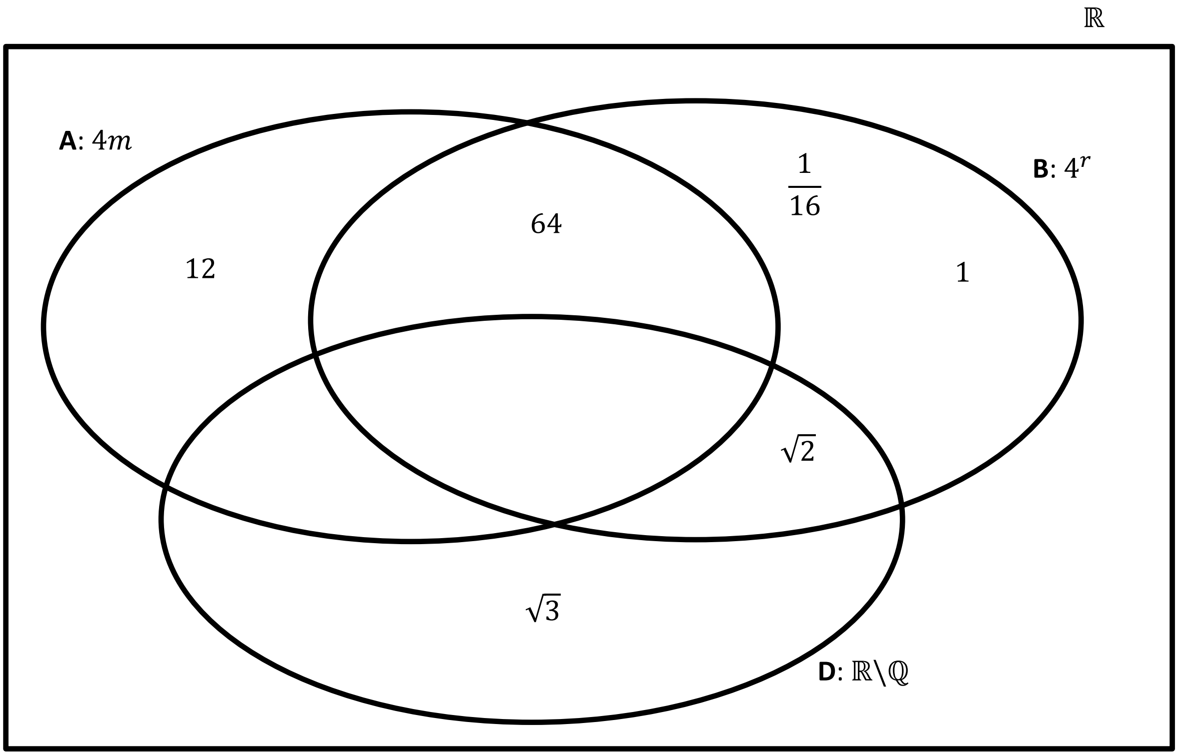
P1 Q4	Model Solution – 30 Marks	Marking Notes
(a)	$\overline{u} = 3 + 7i$ So $2(3 + 7i) + 5i(3 - 7i)$ **... S1** $= 6 + 14i + 15i - 35i^2$ **... S2** $= 6 + 35 + 29i$ $= 41 + 29i$ **... S3**	**Scale 10C (0, 3, 7, 10)** If i^2 is not shown / dealt with, award *Low Partial Credit* at most Consider solution as consisting of 3 steps: **Step 1**: Subs into $2\overline{u} + 5iu$ correctly **Step 2**: Multiplies in 2 and $5i$ (without simplification) **Step 3**: Answer in correct form *Low Partial Credit* • Work of merit, for example, $3 + 7i$, or some correct substitution into $2\overline{u} + 5iu$ *High Partial Credit* • 2 steps correct

P1 Q4	Model Solution – 30 Marks	Marking Notes
(b)	$r = \sqrt{3^2 + \left(\sqrt{3}\right)^2} = \sqrt{12}$ **...S1** $\text{Ref}\,(\theta) = \tan^{-1}\frac{\sqrt{3}}{3} = \tan^{-1}\frac{1}{\sqrt{3}}$ $\text{Ref}\,(\theta) = 30°$ or $\frac{\pi}{6}$ $\theta = 360 - 30 = 330°$ **or** $-30°$ **or** $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$ **or** $-\frac{\pi}{6}$ **...S2** $\left(3 - \sqrt{3}i\right)^8$: $= \left(\sqrt{12}\right)^8[\cos(8(330)°) + i\sin(8(330)°)]$ **...S3** $= 12^4[\cos 120° + i\sin 120°]$ $= 20\,736\left[-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right]$ **...F*** $= -10\,368 + 10368\sqrt{3}\,i$ **...S4**	**Scale 15D (0, 3, 8, 12, 15)** Candidates must engage with polar form or de Moivre's theorem in order to be awarded any credit. Correct answer with no supporting work is awarded *No Credit*. Consider solution as consisting of 4 steps: **Step 1**: Finds r **Step 2**: Finds θ **Step 3**: Applies de Moivre's Theorem **Step 4**: Finishes (answer in rectangular form) Note: de Moivre's Theorem must be applied, either correctly or incorrectly, for Step 4 to be considered correct. *Low Partial Credit* • Work of merit in finding r or θ, or some use of de Moivre's Theorem *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Notes re ** • Apply a * if the answer is in the form $a(b + ci)$ • Apply a * if the imaginary part is rounded rather than exact

P1 Q4	Model Solution – 30 Marks	Marking Notes
(c)	Note that $\lvert a \rvert = 5$ and $\lvert b \rvert = 3$. There are 4 different combinations of a and b, with matching values of θ: • **Same signs, both positive**: $a = 5, b = 3, \theta = 20°$ or $380°$ or ... • **Same signs, both negative**: $a = -5, b = -3, \theta = 20°$ or $380°$ or ... • **Different signs, case 1**: $a = 5, b = -3, \theta = -160°$ or $200°$ or ... • **Different signs, case 2**: $a = -5, b = 3, \theta = -160°$ or $200°$ or ... For example: $a = 5$ **... S1** $b = 3$ **... S2** $15[\cos(160 + \theta) + i\sin(160 + \theta)] = -15$ So $160 + \theta = 180$ **... S3** And $\theta = 20°$ [or $380°$, or ...] **... S4**	**Scale 5D (0, 2, 3, 4, 5)** Accept correct answer without supporting work. Note: $v = \pm 3$ and $w = \pm 5$ is considered work of merit, as a and b are not used. Consider solution as having 4 steps: **Step 1:** finds a value of a **Step 2:** finds a value of b **Step 3:** generates $160 + \theta$ **Step 4:** finds a matching value for θ If Step 3 is **incorrect**, award *Mid Partial Credit* at most. *Low Partial Credit* • Work of merit, for example, some knowledge of the modulus formula, for example, $\sqrt{a^2 + b^2}$ or $w = \pm 5$; **or** $ab(\cos\theta + i\sin\theta)(\cos 160 + i\sin 160)$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct, **including** step 3

P1 Q5	Model Solution – 30 Marks	Marking Notes
(a)	$64 = 4^3$ $\frac{1}{16} = \frac{1}{4^2} = 4^{-2}$ $2 = \sqrt{4} = 4^{1/2}$	**Scale 10C (0, 3, 7, 10)** Accept correct answers without supporting work *Low Partial Credit* • Work of merit, for example, some relevant work with indices *High Partial Credit* • 2 parts correct *Note re *:* • Apply a * once if answer(s) are given as the values of r (i.e. $3, -2, \frac{1}{2}$) instead of in correct form.
(b)	**(i)** *Completed Venn diagram below* **(ii)** *Any valid answer – a rational number that is neither a multiple of* 4 *nor a rational power of* 4. *Note that answer may appear in the answerspace or in the diagram.*	**Scale 10D (0, 2, 5, 8, 10)** Accept correct answers without supporting work Consider solution as having 7 parts: the 6 numbers in **(b)(i)** that must be put in the correct region of the Venn diagram, and the answer to **(b)(ii)**. *Low Partial Credit* • 1 part correct *Mid Partial Credit* • 3 parts correct *High Partial Credit* • 5 parts correct

P1 Q5	Model Solution – 30 Marks	Marking Notes



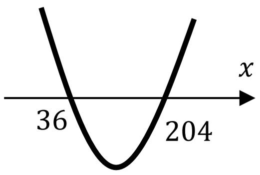
(c)	**Method 1:** $4^k = 9$ so $\log_3(4^k) = \log_3 9$ **... S1** i.e. $k \log_3 4 = 2$ **... S2** so $k = \frac{2}{\log_3 4}$ or $\frac{1}{\log_3 2}$ **... S3** **Method 2:** $4^k = 9$ So $\log_4 9 = k$ **... S2** Then $k = \frac{\log_3 9}{\log_3 4}$ **... S1** $= \frac{2}{\log_3 4}$ **... S3**	**Scale 10C (0, 3, 7, 10)** Consider solution as having 3 steps: **Step 1:** Equation in $\log_3$ **Step 2:** Linear equation in k **Step 3:** finds k in correct form Steps 1 and 2 can be in any order. *Low Partial Credit* • Work of merit, for example, $9 = 3^2$, or some relevant use of logs • 1 step correct *High Partial Credit* • 2 steps correct

P1 Q6	Model Solution – 30 Marks	Marking Notes
(a)	$P(n)$: $n! > 3^n$ for all $n \in \mathbb{N}, n \geq 7$. $\boldsymbol{P(7)}$: Check $P(7)$: $7! = 5040$ $3^7 = 2187$ $[< 5040]$ So $P(7)$ is true. **… S1** $\boldsymbol{P(k)}$: Assume $P(k)$ true [for $k \geq 7$] So $k! > 3^k$ **… S2** $\boldsymbol{P(k+1)}$**, Method 1**: Show $P(k+1)$ true, that is $(k+1)! > 3^{k+1}$ $(k+1)! = (k+1) \times k!$ **… S3** $> (k+1) \times 3^k$ [… by $P(k)$] $> 3 \times 3^k$ … as $k \geq 7$ $= 3^{k+1}$ **… S4** So $P(k+1)$ true. Therefore $P(n)$ is true for all $n \in \mathbb{N}, n \geq 7$. **OR** $\boldsymbol{P(k+1)}$**, Method 2:** Show $P(k+1)$ true, that is $(k+1)! > 3^{k+1}$ $k! > 3^k$ [… by $P(k)$] So $(k+1)(k!) > (k+1)(3^k)$ **… S3** And $(k+1)! > 3(3^k)$ … as $k \geq 7$ $= 3^{k+1}$ **… S4** So $P(k+1)$ true. etc.	**Scale 15D (0, 3, 8, 12, 15)** Consider solution as consisting of 4 steps: **Step 1.** $\boldsymbol{P(7)}$ verified **Step 2.** $\boldsymbol{P(k)}$ stated **Step 3.** $\boldsymbol{P(k+1)}$ stated **and** $\boldsymbol{(k+1) \times k!}$ **Step 4.** $\boldsymbol{P(k+1)}$ proved Steps 1 and 2 can be in any order. Reason for $(k+1) \times 3^k > 3 \times 3^k$, or equivalent, must appear for Step 4 to be correct. *Low Partial Credit* • Work of merit, for example, $P(n)$ checked for $n \neq 7$; **or** $P(k+1)$ stated; **or** states "$P(7)$ true. $P(k)$ true implies $P(k+1)$ true" *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Note re ** • Apply a * if part or all of the conclusion is omitted, but it is otherwise correct. Conclusion has three parts; these do not all have to come at the end of the proof: ○ $P(7)$ true ○ $P(k)$ true implies $P(k+1)$ true ○ $P(n)$ true for all $n \in \mathbb{N}, n \geq 7$ • Apply a * if $n > 7$ is used instead of $n \geq 7$

P1 Q6	Model Solution – 30 Marks	Marking Notes
(b)	**Method 1:** $(n+1)(n) = 156$ $\sqrt{156} = 12{\cdot}48 \ldots$ Then $13 \times 12 = 156$ so $n = 12$ **Method 2:** $n^2 + n - 156 = 0$ $(n-12)(n+13) = 0$ so $n = 12$ [as $n > 0$]	**Scale 5C (0, 2, 3, 5)** Accept correct verified value of n. Some relevant work with factorials required to get any credit. If $n(n+1)$ is not used, then award *Low Partial Credit* at most, unless the correct value of n is verified. *Low Partial Credit* • Work of merit, for example, indicates $n(n+1)$, or trials one value of n fully. *High Partial Credit* • Significant work of merit, for example, $(n+1)n = 156$, or indicates $n(n+1)$ **and** finds $\sqrt{156}$ *Note re ** • Apply a * if 12 and -13 are both found, but 12 isn't selected

P1 Q6	Model Solution – 30 Marks	Marking Notes
(c)	$r < 0$, as $S_3 < S_2$ Also, $T_3 < 0$, for the same reason. So $T_2 > 0$ and $T_1 < 0$. Pick $T_1 < 0$. For example, $T_1 = -1$ **... S1** Then $T_2 + T_1 = 2$, so $T_2 = 2 - T_1 = 3$ **... S2** Then $r = \frac{T_2}{T_1} = \frac{3}{-1} = -3$ so $T_3 = -3 \times 3 = -9$ **... F*** and $S_3 = 2 + T_3 = -7$ **... S3**	**Scale 10C (0, 3, 7, 10)** Accept correct answer without supporting work. Consider solution as consisting of 3 steps: **Step 1.** $\boldsymbol{T_1}$ found, with $\boldsymbol{T_1 < 0}$ **Step 2**. $\boldsymbol{T_2}$ found, with $\boldsymbol{T_1 + T_2 = 2}$ **Step 3**. $\boldsymbol{T_3}$ and $\boldsymbol{S_3}$ found, with $\boldsymbol{T_1 < 0}$ Step 2 can be considered correct even if $T_1 > 0$. If T_3 is found correctly (relative to candidate's work) but S_3 is not, then consider Step 3 as correct with a *. *Low Partial Credit* • Work of merit, for example, states a **and** ar, or produces three terms in geometric sequence *High Partial Credit* • 2 steps correct *Note re ** • Apply a * if T_1, T_2, and T_3 are found, but S_3 is incorrect or not found.

P1 Q7	Model Solution – 50 Marks	Marking Notes
(a)	$V(60) = 0{\cdot}025(60^3) - 9(60^2) + 550{\cdot}8(60) + 68900$ $= 74\,948$ cm^3	**Scale 5B (0, 2, 5)** Accept correct answer without supporting work. Treat setting $t = 1$ as an error rather than a slip, as it simplifies the work. *Partial Credit* • $V(60)$ fully substituted • Error(s) in substituting $V(60)$, but evaluates correctly *Note re ** • Apply a * for missing / incorrect unit
(b)	$V'(t) = 0{\cdot}075t^2 - 18t + 550{\cdot}8$ **…S1** $V'(150) = 0{\cdot}075(150^2) - 18(150) + 550{\cdot}8$ **…S2** $= -461{\cdot}7$ [cm^3 / second] **… S3**	**Scale 10C (0, 3, 7, 10)** Accept correct answer with incorrect or no unit. Some differentiation required to get any credit Consider solution as consisting of 3 steps: **Step 1.** Finds $\boldsymbol{V'(t)}$ **Step 2**. $\boldsymbol{V'(150)}$ fully substituted **Step 3**. Evaluates $\boldsymbol{V'(150)}$ *Low Partial Credit* • Work of merit, for example, some correct differentiation *High Partial Credit* • 2 steps correct *Note re ** • Apply a * if the direction of the rate of change is not included (if the minus sign does not appear, and no mention of "decreasing")

P1 Q7	Model Solution – 50 Marks	Marking Notes
(c)	$V'(t) < 0$ **…S1** so $0{\cdot}075t^2 - 18t + 550{\cdot}8 < 0$ Roots: $t = \frac{18 \pm \sqrt{18^2 - 4(0{\cdot}075)(550{\cdot}8)}}{2(0{\cdot}075)}$ **…S2** $t = 36$ and $t = 204$ **…S3**  Answer: $t \in (36, 204)$ **…S4** or $36 < t < 204$ or $t > 36$ and $t < 204$ *or equivalent*	**Scale 10D (0, 2, 5, 8, 10)** Accept “between 36 and 204” Accept correct answer without supporting work Consider solution as consisting of 4 steps: **Step 1.** Indicates that $\boldsymbol{V'(t) < 0}$ or $\boldsymbol{= 0}$ **Step 2**. Quadratic formula fully substituted, or quadratic equation factorised **Step 3**. 2 roots found **Step 4**. Correct solution Note: Assume Step 1 is done if Step 2 is completed. *Low Partial Credit* • Work of merit, for example, brings down $V'(t)$ from (b) *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Notes re* * • Apply a * for endpoints of inequality included: $t \in [36, 204]$, or equivalent (for example, “from 36 to 204” or “36 to 204”). • Apply a * if $t > 36$ and $t < 204$ presented, but no indication what to do with these (i.e. intersect the sets)

P1 Q7	Model Solution – 50 Marks	Marking Notes
(d)	$V = \pi r^2 h = \pi(26^2)h = 676\pi\ h$ $\frac{dV}{dh} = 676\pi$ [cm³/cm] **… S1** **Rate of change – method 1**: $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ so $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh}$ **… S2** At $t = 150$, $\frac{dV}{dt} = -461{\cdot}7$ (from part **(b)**) $\frac{dh}{dt} = -461.7 \div 676\pi$ **…S3** $= -0{\cdot}22$ [cm/sec] [2 D.P.] **…S4** **Rate of change – method 2**: $\frac{dV}{dh} = 676\pi$ [cm³/cm] **… S1** With $h(t)$ as the height at time t: $V(t) = \pi(26^2)h(t) = 676\pi\ h(t)$ **… S2** So $V'(t) = 676\pi\ h'(t)$ and $h'(t) = \frac{V'(t)}{676\pi}$ so $h'(150) = \frac{V'(150)}{676\pi}$ $= \frac{-461{\cdot}7}{676\pi}$ **… S3** $= -0{\cdot}22$ [cm/sec] [2 D.P.] **… S4**	**Scale 10D (0, 2, 5, 8, 10)** Accept correct answers with incorrect or no unit. Consider solution as consisting of 4 steps: **Step 1.** Finds $\boldsymbol{\frac{dV}{dh}}$ **Step 2.** $\boldsymbol{\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}}$ or equivalent / $\boldsymbol{V(t) = \pi(26^2)h(t)}$ **Step 3.** $\boldsymbol{\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}}$ fully substituted **Step 4.** Finds $\boldsymbol{h'(150)}$ *Low Partial Credit* • Work of merit, for example, $\pi(26^2)h$ or $\frac{1}{3}\pi(26^2)h$; **or** indicates that $\frac{dV}{dt}$ = candidate's answer from **(b)**; **or** writes down a relevant derivative other than $\frac{dV}{dh}$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Note re ** • Incorrect rounding, otherwise correct.

<table>
<tr><th>P1 Q7</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(e)</td><td>
1. Finding value of h: Method 1:
Local minimum when $16\pi h - \pi h^2$ has a local maximum; differentiating gives:
$D'(h) = 16\pi - 2\pi h$ … S1
$16\pi - 2\pi h = 0$ … S2
so $h = 8$ … S3

1. Finding value of h: Method 2 (chain rule):
$C(h) = 20(16\pi h - \pi h^2)^{-1}$
$C'(h) = -20(16\pi h - \pi h^2)^{-2}(16\pi - 2\pi h)$ …S1
$= \frac{-20(16\pi - 2\pi h)}{(16\pi h - \pi h^2)^2} = 0$ at minimum … S2
so $16\pi - 2\pi h = 0$
and $h = 8$ … S3

1. Finding value of h: Method 3 (quotient rule):
$C'(h) = \frac{(16\pi h - \pi h^2)(0) - 20(16\pi - 2\pi h)}{(16\pi h - \pi h^2)^2}$ … S1
$= \frac{-20(16\pi - 2\pi h)}{(16\pi h - \pi h^2)^2} = 0$ at minimum … S2
so $16\pi - 2\pi h = 0$
and $h = 8$ … S3

2. Finding local minimum value:
$C(8) = \frac{20}{16\pi(8) - \pi(8^2)}$
$= 0{\cdot}09 \ldots$
$= 0{\cdot}1$ [cm/sec] [1 D.P.] … S4
</td><td>
Scale 10D (0, 2, 5, 8, 10)
Accept correct answer with incorrect or no unit.

Consider solution as consisting of 4 steps:
Step 1. Carries out all necessary differentiation
Step 2. Sets relevant derivative $= \mathbf{0}$
Step 3. Finds value of $\boldsymbol{h}$
Step 4. Finds value of $\boldsymbol{C}$

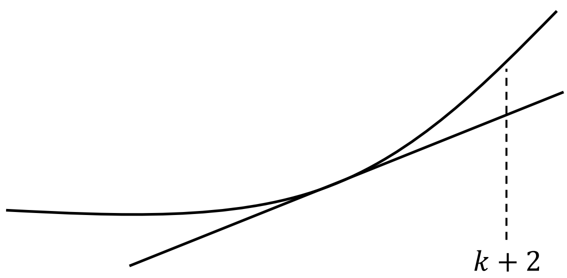
Low Partial Credit
• Work of merit, for example, some correct differentiation
Mid Partial Credit
• Fully correct differentiation (unsimplified)
• 2 steps correct
High Partial Credit
• 3 steps correct
Note re *
• Apply a * for incorrect rounding
</td></tr>
</table>

P1 Q7	Model Solution – 50 Marks	Marking Notes
(f)	$C(h)$ not defined when $16\pi h - \pi\, h^2 = 0$ i.e. $h(16 - h) = 0$ so $h = 0$ or $h = 16$ i.e. $M = 16$	**Scale 5B (0, 2, 5)** Accept correct answer without supporting work Accept $15\cdot\dot{9}$ *Partial Credit* • Work of merit, for example, sets $16\pi h - \pi\, h^2 = 0$, or states $C(h)$ is not defined at $h = 0$, or trials 2 or more values of h *Notes re* * • Apply a * for $0 < h < 16$ or for $h = 16$ • Apply a * for "$M < 16$"

P1 Q8	Model Solution – 50 Marks	Marking Notes
(a)	$F(11) = 440 \times 2^{\frac{11}{12}}$ $= 830{\cdot}6\ldots$ $= 831$ [Hz] [$\in \mathbb{N}$]	**Scale 5B (0, 2, 5)** Accept correct answer without supporting work Note: must sub in 11 in order to be awarded any credit Accept correct answer with incorrect or no unit. *Partial Credit* • Work of merit, for example, correct substitution into $F(11)$ *Note re ** • Apply a * for incorrect rounding • Apply a * for a fully substituted formula and an answer of 830
(b)	$F(-6) = 440 \times 2^{-\frac{6}{12}}$ $= 311{\cdot}126\ldots$ $= 311{\cdot}1$ [Hz] [4 sig figs]	**Scale 10C (0, 3, 7, 10)** Accept correct answer without supporting work Note: 6 or -6 must appear in substitution for x in order to be awarded any credit Accept correct answer with incorrect or no unit. *Low Partial Credit* • Work of merit, for example, 6 substituted for x and no further work; **or** $a - 6$ is substituted for x and evaluated, where $a \in \mathbb{N}$ is a constant *High Partial Credit* • $F(-6)$ fully substituted • $F(6)$ evaluated correctly *Note re ** • Apply a * for incorrect rounding

P1 Q8	Model Solution – 50 Marks	Marking Notes
(c)	**Method 1:** $440 \times 2^{\frac{x}{12}} = 660$ $\Leftrightarrow \quad 2^{\frac{x}{12}} = \frac{660}{440} = 1{\cdot}5$ **... S1** $\Leftrightarrow \quad \ln\left(2^{\frac{x}{12}}\right) = \ln 1{\cdot}5$ **... S2** $\Leftrightarrow \quad \frac{x}{12} \ln 2 = \ln 1{\cdot}5$ $\Leftrightarrow \quad x = \frac{12 \ln 1{\cdot}5}{\ln 2}$ **... S3** $= 7{\cdot}019 \ldots$ **... F*** $= 7{\cdot}02$ [2 D.P.] **... S4** **Method 2:** $440 \times 2^{\frac{x}{12}} = 660$ $\Leftrightarrow \quad 2^{\frac{x}{12}} = \frac{660}{440} = 1{\cdot}5$ **... S1** $\Leftrightarrow \quad \log_2 1{\cdot}5 = \frac{x}{12}$ **... S2** $\Leftrightarrow \quad x = 12 \log_2 1{\cdot}5$ **... S3** $= 7{\cdot}019 \ldots$ **... F*** $= 7{\cdot}02$ [2 D.P.] **... S4**	**Scale 10D (0, 2, 5, 8, 10)** Correct answer with no supporting work is awarded *No Credit* Accept fully verified correct answer (i.e. two numbers that round to $7{\cdot}02$ substituted in for x, with $\text{Ans}_1 < 660$ and $\text{Ans}_2 > 660$, **and** correct answer) Consider solution as consisting of 4 steps: **Step 1.** Isolates $\mathbf{2^{\frac{x}{12}}}$ **Step 2**. One relevant use of logs **Step 3**. Isolates $\boldsymbol{x}$ **Step 4**. Evaluates $\boldsymbol{x}$ *Low Partial Credit* • Work of merit, for example, sets up the correct equation; **or** substitutes in at least two values for x *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct • Substitutes in $7{\cdot}01$ and $7{\cdot}02$ for x, **and** has correct answer *Note re ** • Apply a * for incorrect rounding

P1 Q8	Model Solution – 50 Marks	Marking Notes
(d)	$F'(x) \quad = 440 \times 2^{\frac{x}{12}} \times \ln 2 \times \frac{1}{12}$ **or** $\frac{110 \ln 2}{3} \times 2^{\frac{x}{12}}$	**Scale 5C (0, 2, 3, 5)** Accept correct answer without supporting work Consider solution as requiring 4 terms: 440, $2^{\frac{x}{12}}$**,** $ln\, 2$, and $\frac{1}{12}$ *Low Partial Credit* • $\ln 2$ or $\frac{1}{12}$ *High Partial Credit* • 3 terms correct

P1 Q8	Model Solution – 50 Marks	Marking Notes
(e)	**(i)** $G(k+2) \quad = 197 + 2 \times 5{\cdot}8$ $\qquad\qquad\quad = 208{\cdot}6$ **OR** *Let* $y = g(x)$ *be the tangent at* $\big(k, G(k)\big)$*:* $g(x) = \int 5{\cdot}8 \;\, \mathrm{d}x = 5{\cdot}8x + c$ [for a constant c] So $\qquad g(k+2) - g(k) = 11{\cdot}6$ And $\qquad g(k+2) = 11{\cdot}6 + 197 = 208{\cdot}6$ **(ii)** The estimate is too small. *Any valid justification, including through diagram, for example:* The graph curves upwards, so it [the graph of the actual value] is always above the tangent line. **OR**  **OR** $\qquad G'(k) = bAe^{bk} = 5{\cdot}8$ So $\qquad 197b = 5{\cdot}8 \qquad\qquad$ ... as $Ae^{bk} = 197$ Then $\quad b = \frac{29}{985}$ So $\quad G(k+2) = Ae^{b(k+2)}$ $\qquad\qquad\qquad = Ae^{bk} \times e^{2b}$ $\qquad\qquad\qquad = 197 \times e^{\frac{58}{985}}$ $\quad$ ... as $Ae^{bk} = 197$ $\qquad\qquad\qquad = 208{\cdot}9$ ...	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit* • Work of merit in 1 part, for example: in **(i)**, draws a diagram of a [tangent] line, **or** $197 + 5{\cdot}8$ or $2 \times 5{\cdot}8$, **or** indicates integration • in **(ii)**, draws a diagram of an exponential curve, or correct answer • in **(ii)**, some correct differentiation of $G(x)$ *Mid Partial Credit* • Work of merit in both parts • One part correct *High Partial Credit* • One part correct **and** work of merit in the other part

P1 Q8	Model Solution – 50 Marks	Marking Notes
(f)	**Method 1: simultaneous equations, for example:** ① $\mathbf{S} + \mathbf{A} + \mathbf{T} + \mathbf{B} = 384$ ② $2\,\mathbf{S} = \mathbf{A}$ ③ $\mathbf{T} = \mathbf{B} + 12$ ④ $\mathbf{S} + \mathbf{A} = 3(\,\mathbf{T} + \mathbf{B}\,)$ **... S1** ①,④: $3(\,\mathbf{T} + \mathbf{B}\,) + \mathbf{T} + \mathbf{B} = 384$ **... S2** so $4\mathbf{T} + 4\mathbf{B} = 384$ and ③: $4(\mathbf{B} + 12) + 4\mathbf{B} = 384$ so $8\mathbf{B} + 48 = 384$ and $\mathbf{B} = \frac{336}{8} = 42$ **... S3** So $\mathbf{T} = \mathbf{B} + 12 = 42 + 12 = 54$ ④: $\mathbf{S} + \mathbf{A} = 3(54 + 42) = 288$ and ②: $\mathbf{S} + 2\mathbf{S} = 288$ so $\mathbf{S} = \frac{288}{3} = 96$ and ②: $\mathbf{A} = 2\mathbf{S} = 2(96) = 192$ **...S4** $\mathbf{S} = 96$; $\mathbf{A} = 192$; $\mathbf{T} = 54$; $\mathbf{B} = 42$ **OR** **Method 2: Using (mainly) ratio:** $\mathbf{S} + \mathbf{A}$ is 3 times $\mathbf{T} + \mathbf{B}$, so split 384 in ratio 3:1. $384 \div 4 = 96 = \mathbf{T} + \mathbf{B}$ **... S1** $96 \times 3 = 288 = \mathbf{S} + \mathbf{A}$ **... S3** **A** is twice **S**, so split 288 in ratio 2:1. $288 \div 3 = 96 = \mathbf{S}$ $96 \times 2 = 192 = \mathbf{A}$ **... S4** **T** is 12 bigger than **B**, so $\frac{96-12}{2} = 42 = \mathbf{B}$ and so $42 + 12 = 54 = \mathbf{T}$ **... S2**	**Scale 15D (0, 3, 8, 12, 15)** Consider **Method 1** solution as consisting of 4 steps: **Step 1.** Writes down 4 correct equations (① – ④) **Step 2**. Uses two equations together **Step 3**. Finds one of **S**, **A**, **T**, **B** **Step 4**. Finds three remaining values Consider **Method 2** solution as consisting of 4 steps: **Step 1.** Finds **T** + **B** **Step 2**. Finds **T** and finds **B** **Step 3**. Finds **S** + **A** **Step 4**. Finds **S** and finds **A** *Low Partial Credit* • Work of merit, for example: one correct equation, **or** some work towards forming an equation, **or** indicates ratio 3: 1 or 2: 1 *Mid Partial Credit* • Four correct equations • 2 steps correct *High Partial Credit* • 3 steps correct

<table>
<tr><th>P1 Q9</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a) (b)</td><td>

(a)

<table>
<tr><td>$20, 40, 20$</td><td>3</td><td>80</td></tr>
<tr><td>$20, 40, 60, 40, 20$</td><td>5</td><td>$\mathbf{180}$</td></tr>
<tr><td>$20, 40, 60, 80, 60, 40, 20$</td><td>$\mathbf{7}$</td><td>$320$</td></tr>
<tr><td>$\mathbf{20, 40, 60, 80, 100, 80, 60, 40, 20}$</td><td>$\mathbf{9}$</td><td>$\mathbf{500}$</td></tr>
</table>

(b)

[$12 < 15$, so this is before half way]

$12 \times 20 = 240$ m

</td><td>

Scale 10D (0, 2, 5, 8, 10)

Accept correct answer without supporting work.

Consider solution as requiring 6 parts: the five entries in the table, and part (b)

Low Partial Credit

- Work of merit, for example,
1 of the 6 parts correct, or
3 terms correct in the shuttles of Set 4

Mid Partial Credit

- 3 (or 4) parts correct

High Partial Credit

- 5 parts correct

Note re *

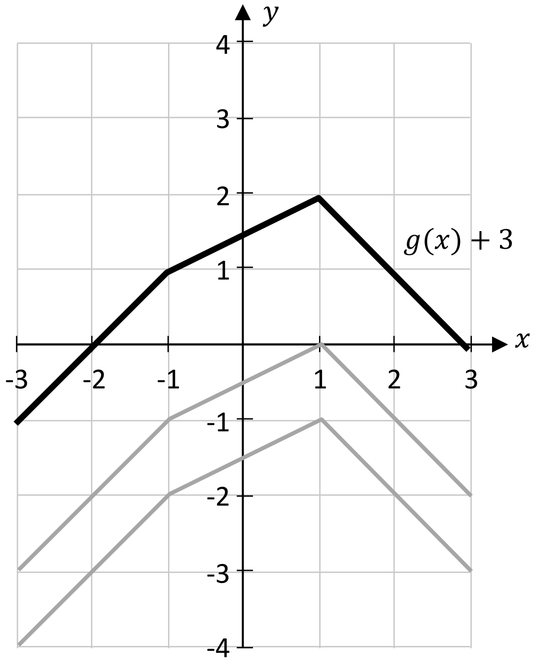
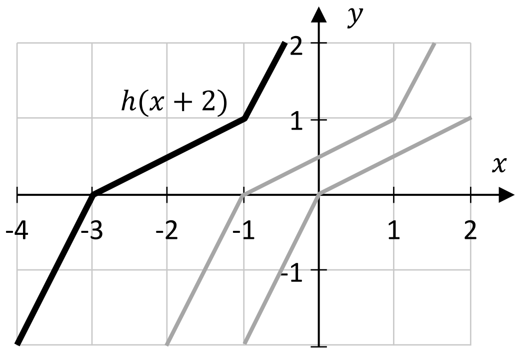
- Apply a * for incorrect or missing unit in part (b)

</td></tr>
</table>

P1 Q9	Model Solution – 50 Marks	Marking Notes
(c)	**Method 1**: $D = 2\{20 + 40 + \cdots + 20n\} + 20(n+1)$ **…S1** $= 2\left\{\frac{n}{2}[2(20) + (n-1)20]\right\} + 20n + 20$ **…S2** $= \{40n + 20n^2 - 20n\} + 20n + 20$ **…S3** $= 20n^2 + 40n + 20$ **…S4** **Method 2**: ① $n = 1$: $a + b + c = 80$ ② $n = 2$: $4a + 2b + c = 180$ ③ $n = 3$: $9a + 3b + c = 320$ **…S1** ② – ①: $3a + b = 100$ ④ ③ – ②: $5a + b = 140$ ⑤ **…S2** ⑤ – ④: $2a = 40$ So $a = 20$ **…S3** Then ④: $b = 40$ And ①: $c = 20$ **…F*** So answer: Dist. $= 20n^2 + 40n + 20$ **…S4** **Method 3**: 1st differences: 100, 140 2nd difference: $40 = 2a$ **…S1** So $a = 20$. **…S2** ① $n = 1$: $b + c = 60$ … as $a = 20$ ② $n = 2$: $2b + c = 100$ … as $a = 20$ **…S3** ② – ①: $b = 40$ So ①: $c = 20$ **…F*** So answer: Dist. $= 20n^2 + 40n + 20$ **…S4**	**Scale 10D (0, 2, 5, 8, 10)** Consider **Method 1** solution as consisting of 4 steps: **Step 1.** Splits up the sum in into two or three smaller linear sums **Step 2**. Fully substituted closed form **Step 3**. Brackets all multiplied out **Step 4**. Answer in correct form Consider **Method 2** solution as consisting of 4 steps: **Step 1.** Three equations in $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ **Step 2**. Two equations in (the same) two unknowns **Step 3**. Finds one constant **Step 4**. Finds other two constants, and answer in correct form Consider **Method 3** solution as consisting of 4 steps: **Step 1.** Finds second differences **Step 2**. Finds $\boldsymbol{a}$ **Step 3**. Two equations in $\boldsymbol{b}$ and $\boldsymbol{c}$ **Step 4**. Finds other two constants, and answer in correct form *Low Partial Credit* • Work of merit, for example: one correct equation, or indicates 1st differences. *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Note re ** • In Method 2 or Method 3, apply a * if a, b, and c are found, but answer is not in the required form ($an^2 + bn + c$)

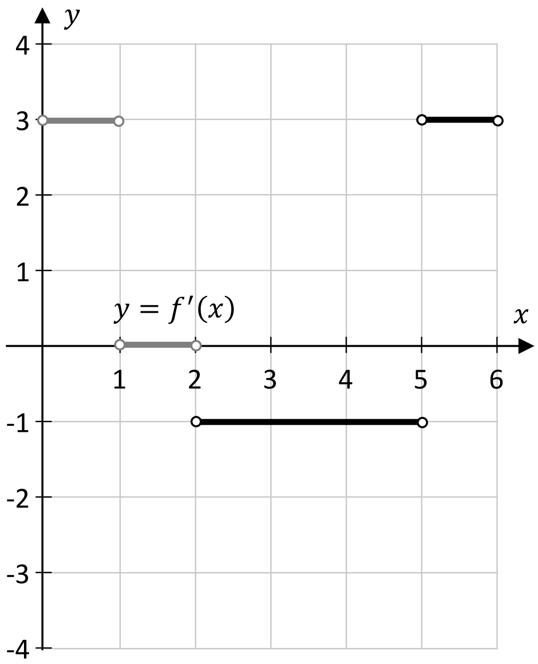
P1 Q9	Model Solution – 50 Marks	Marking Notes
(d)	Middle shuttle is number $\frac{43+21}{2} = 32$ So $\quad h = 31$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit, for example, finds $43 - 21$, or indicates middle value, or 63 with supporting work *High Partial Credit* • Finds 32 with supporting work
(e)	$k > 18$, so $T_k = a - 20k$ $T_{19} = 19 \times 20 = 380 = a - 20(19)$ So $\quad a = 760$ and $T_k = 760 - 20k$ **OR** $T_k \quad = 380 + (k - 1 - 18)(-20)$ **or** $\quad 380 - 20(k - 19)$ **OR** $a + 18d = 380$ $\underline{a + 19d = 360}$ $d = -20$ and $a = 740$ $T_k = 740 + (k - 1)(-20)$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit, for example: finds $20k$ • Finds T_{19} *High Partial Credit* • $a - 20(19) = 380$ • $a + 18d = 380$
(f)	$\left\lvert w\left(\frac{5+30}{30}\right) - w\left(\frac{36}{37-5}\right)\right\rvert$ $= \quad w \times \left\lvert \frac{35}{30} - \frac{36}{32} \right\rvert$ $= \quad \frac{w}{24}$	**Scale 10C (0, 3, 7, 10)** Accept $-\frac{w}{24}$ *Low Partial Credit* • Work of merit, for example, r filled into one formula (with or without w) *High Partial Credit* • Both formulae substituted *Note re ** • Apply a * if the answer is the coefficient of w • Apply a * if the fraction is not simplified

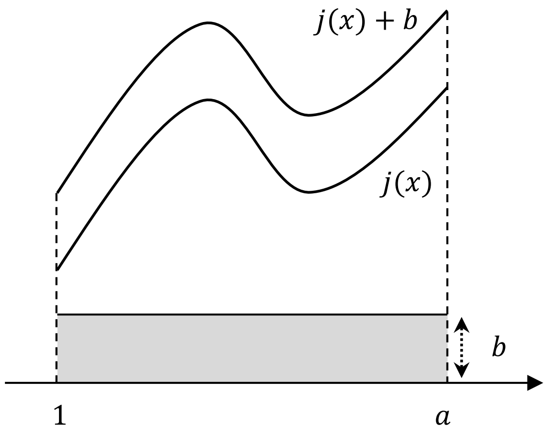
P1 Q9	Model Solution – 50 Marks	Marking Notes
(g)	$w\left(\frac{r+30}{30}\right) = w\left(\frac{36}{37-r}\right)$ so $(37-r)(r+30) = 30 \times 36$ **... S1** so $37r + 1110 - r^2 - 30r = 1080$ i.e. $r^2 - 7r - 30 = 0$ **... S2** so $(r-10)(r+3) = 0$ **... S3** i.e. $r = 10$ [*as* $r \geq 0$] **... S4**	**Scale 10D (0, 2, 5, 8, 10)** Accept correct answer if fully verified. Otherwise, correct answer without supporting work is awarded *Low Partial Credit*. Accept where a non-zero value is subbed in for w and the equation in r is solved correctly. Consider solution as consisting of 4 steps: **Step 1.** Forms equation without $\boldsymbol{r}$ in denominator **Step 2**. Expresses in directly solvable form (for example, in form $\boldsymbol{ar^2 + br + c = 0}$, for constants $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$) **Step 3**. Factorises quadratic expression / fully substituted quadratic formula **Step 4**. Solves for $\boldsymbol{r}$ *Low Partial Credit* • Work of merit, for example, first line of solution, or trials a value other than $r = 5$ in both formulae *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Note re ** • Apply a * if $r = 10$ and $r = -3$ are found, but $r = 10$ is not selected.

P1 Q10	Model Solution – 50 Marks	Marking Notes
(a)	**(i)**  **(ii)**  	**Scale 10C (0, 3, 7, 10)** Accept correct answers without supporting work. *Low Partial Credit* • Work of merit, for example, in **(i)**, shifts graph or segment of graph vertically an incorrect number of units in **(ii)**, shifts graph or segment of graph horizontally an incorrect number of units *High Partial Credit* • 1 part correct (**(i)** or **(ii)**) • Work of merit in both parts *Notes re ** • Apply a * once if multiple graphs drawn, but correct graph is not labelled • Apply a * in each section if the endpoint of one segment is incorrect, and the rest of the graph is correct • Apply a * if the graph of $g(x)+1$ is translated vertically upwards 3 units • Apply a * if the graph of $h(x-1)$ is translated horizontally to the left 2 units

P1 Q10	Model Solution – 50 Marks	Marking Notes
(b)	**(i)** 5 **(ii)** *Any three* ***non-zero*** *multiples of* 5*, for example:* $5, 20, -10$	**Scale 5C (0, 2, 3, 5)** Accept correct answers without supporting work. Accept period of -5 *Low Partial Credit* • Work of merit, for example, in **(i)**, multiple of 5; in **(ii)**, 1 value correct *High Partial Credit* • **(i)** correct **and** one value correct in **(ii)** • **(ii)** correct *Note re ** • Apply a * if period is given as an interval of length 5

P1 Q10	Model Solution – 50 Marks	Marking Notes
(c)	*Here, "area" is taken to mean "signed area".* **Method 1**: **Area from $\boldsymbol{x = 0}$ to $\boldsymbol{x = 1}$**: Area $\left(0 \text{ to } \frac{1}{3}\right)$ below $-1 = -1 \times$ Area $\left(\frac{2}{3} \text{ to } 1\right)$, So Area $(0 \text{ to } 1) =$ Area $\left(0 \text{ to } \frac{2}{3}\right)$ above -1 $= -0{\cdot}5$ **Area from $\boldsymbol{x = 1}$ to $\boldsymbol{x = 3}$**: Area $= 1 + \frac{1}{2} = 1{\cdot}5$ So **total** area $= \int_0^3 f(x)\ dx = 1{\cdot}5 - 0{\cdot}5 = 1$ **Method 2:** $\int_0^1 [3x-2]\,dx + \int_1^2 [1]\,dx + \int_2^3 [3-x]\,dx$ $= \left[\frac{3x^2}{2} - 2x\right]_{x=0}^{x=1} + 1 + \left[3x - \frac{x^2}{2}\right]_{x=2}^{x=3}$ $= \frac{3}{2} - 2 + 1 + 3(3) - \frac{3^2}{2} - \left(3(2) - \frac{2^2}{2}\right)$ $= -\frac{1}{2} + 1 + \frac{9}{2} - 4$ $= 1$	**Scale 10D (0, 2, 5, 8, 10)** Consider solution as consisting of 4 steps: **Step 1.** Finds area of one region relevant to signed area from $\mathbf{0}$ to $\mathbf{1}$ / finds equation $\boldsymbol{y = 3x - 2}$ **Step 2**. Finds signed area from $\mathbf{0}$ to $\mathbf{1}$ **Step 3**. Finds signed area from $\mathbf{1}$ to $\mathbf{2}$ or finds signed area from $\mathbf{2}$ to $\mathbf{3}$ **Step 4**. Finds total signed area Note: there needs to be some work with negative area in order for Step 4 to be considered correct. *Low Partial Credit* • Work of merit, for example, finds the x intercept $\left(\frac{2}{3}, 0\right)$, or shows understanding of integration, or identifies relevant region *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

P1 Q10	Model Solution – 50 Marks	Marking Notes
(d)		**Scale 5C (0, 2, 3, 5)** Accept correct answer without supporting work No penalty is applied for mishandling graph at $x = 2, 5$, or 6 *Low Partial Credit* • Work of merit, for example, any vertical translation of $[(2, -1), (5, -1)]$ or $[(5, 3), (6, 3)]$ (needs to be a translation of the whole line segment); or a correct derivative calculated *High Partial Credit* • 1 portion fully correct (for $x \in (2, 5)$ or for $x \in (5, 6)$)

P1 Q10	Model Solution – 50 Marks	Marking Notes
(e)	**Method 1:** $\int_1^a [\, j(x) + b \,]\, dx$ $= \int_1^a j(x)\, dx + \int_1^a b\, dx$ $= 200 + \int_1^a b\, dx$ **... S1** $= 200 + (bx)\Big\vert_{x=1}^{x=a}$ **... S2** $= 200 + ba - b$ **or** $200 + (a-1)b$ **... S3** **Method 2:** Area $= 200 +$ area rectangle **... S1** $= 200 + b(a-1)$ **... S2 & S3**  	**Scale 5D (0, 2, 3, 4, 5)** Accept correct answer without supporting work Consider **Method 1** of solution as consisting of 3 steps: **Step 1**. Replaces $\boldsymbol{\int_1^a j(x)\, dx}$ with $\mathbf{200}$ **Step 2**. Finds antiderivative of $\boldsymbol{b}$ (i.e., $\boldsymbol{bx}$) **Step 3**. Subs in limits for $\boldsymbol{x}$ (must have $\boldsymbol{bx}$) Consider **Method 2** of solution as consisting of 3 steps: **Step 1**. Indicates $\mathbf{200} +$ additional area **Step 2**. Indicates height of rectangle **or** shows $\mathbf{1}$ and $\boldsymbol{a}$ **or** states $\boldsymbol{a - 1}$ **Step 3**. Finds required area. *Low Partial Credit* • Work of merit, for example, diagram of positive $j(x)$ drawn, or shows some understanding of integration *Mid Partial Credit* • 1 step correct *High Partial Credit* • 2 steps correct

P1 Q10	Model Solution – 50 Marks	Marking Notes
(f)	**(i)** $ax^2 + 2ax + a + bx + b + c$ $= ax^2 + bx + c + 2$ $(2a + b)x + a + b + c = bx + c + 2$ ***x* terms**: $2a + b = b$ **… S1** $\Rightarrow a = 0$ **constant terms**: $a + b + c = c + 2$ **...S2** so $0 + b = 2 \Rightarrow b = 2$ **… S3** **(ii)** It must be a [straight] line with a slope of 2 **… S4**	**Scale 15D (0, 3, 8, 12, 15)** Accept if two values are substituted for $\boldsymbol{x}$ and the resulting simultaneous equations in $\boldsymbol{a}$ and $\boldsymbol{b}$ are solved If h is treated as a variable instead of a function, award *Low Partial Credit* at most. Consider solution as consisting of 4 steps: **Step 1.** Equates $\boldsymbol{x}$ terms / one correct equation in $\boldsymbol{a}$ and $\boldsymbol{b}$ **Step 2.** Equates constant terms / second correct equation in $\boldsymbol{a}$ and $\boldsymbol{b}$ **Step 3.** Finds $\boldsymbol{a}$ and $\boldsymbol{b}$ **Step 4.** **(ii)** correct *Low Partial Credit* • Work of merit, for example, first line of solution; **or** writes $h(x+1) =$ $a(x+1)^2 + b(x+1) + c$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Note re ** • Apply a * in **(ii)** if answer is “line” (no mention of what the slope must be) **or** if answer is “slope of 2” (no mention that it is a line)

Structure of the marking scheme – Paper 2

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled **A** divide candidate responses into two categories (correct and incorrect), scales labelled **B** divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	**A**	**B**	**C**	**D**
Number of categories	2	3	4	5
5-mark scales	0, 5	0, 2, 5	0, 2, 3, 5	0, 2, 3, 4, 5
10-mark scales			0, 3, 7, 10	0, 2, 5, 8, 10
15-mark scale				0, 3, 8, 12, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response
- correct response

B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

Model Solutions & Marking Notes – Paper 2

<table>
<tr><th>P2 Q1</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>[20 data points, so median splits into top 10 and bottom ten.

Quartiles split into 4 groups of 5.]

Median: halfway between 10th and 11th, so $= 63{\cdot}5$

Range: $85 - 54 = 31$

IQR:

Q1: halfway between 5th and 6th, so $= \frac{57+57}{2} = 57$

Q3: halfway between 15th and 16th, so $= \frac{69+70}{2} = 69{\cdot}5$

Then IQR $= 69{\cdot}5 - 57 = 12{\cdot}5$</td><td>Scale 10D (0, 2, 5, 8, 10)

Consider solution as consisting of 4 steps:
Step 1. Finds median
Step 2. Finds range
Step 3. Finds Q1 or Q3
Step 4. Finds remaining quartile (Q1 or Q3) and IQR

Low Partial Credit
• Work of merit, for example, indicates that median is middle; or states IQR = Q3 - Q1; or mentions 25% or 75%

Mid Partial Credit
• 2 steps correct

High Partial Credit
• 3 steps correct
• Uses 5th, 10th, 15th value, otherwise correct (Median = 63, Q1 = 57 and Q3 = 69)

Note re *
• Apply a * once if the range and/or the IQR are given as an interval instead of a value</td></tr>
</table>

<table>
<tr><th>P2 Q1</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(b)</td><td>Each hour $= \frac{360}{20} = 18°$

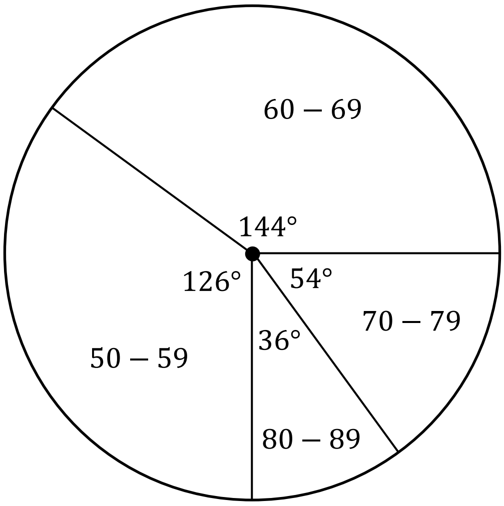
$50 - 59$: $7 \times 18 = 126°$

$60 - 69$: $8 \times 18 = 144°$

$70 - 79$: $3 \times 18 = 54°$

$80 - 89$: $2 \times 18 = 36°$

Pie chart drawn within tolerance, with labels and angles marked; for example:


</td><td>Scale 10D (0, 2, 5, 8, 10)

Note: Only need to calculate 3 angles.

Accept correct pie chart drawn with 3 sectors labelled and the size of 3 angles shown on diagram.

Tolerance of $\pm 3°$

Consider solution as consisting of 4 steps:
Step 1. Calculates 1 angle
Step 2. Calculates remaining 2 angles
Step 3. Draws 1 sector
Step 4. Draws and labels remaining sectors

Low Partial Credit
• Work of merit, for example, one correct relevant calculation indicated; mentions 360°

Mid Partial Credit
• 2 steps correct

High Partial Credit
• 3 steps correct
• 2 angles calculated correctly, and 2 sectors correctly drawn

Note re *
• Apply a * once if more than one label and/or more than one angle is missing from the pie chart</td></tr>
<tr><td>(c)</td><td>$\frac{3+k}{20+k} = \frac{32}{100}$

so $300 + 100k = 640 + 32k$

so $68k = 340$

and $k = 5$</td><td>Scale 10C (0, 3, 7, 10)
Note: Correct value of $\boldsymbol{k}$ verified, award full marks

Low Partial Credit
• Work of merit, for example, $3 + k$ or $20 + k$, or trials one value for k, or mentions 0·32 or 0·68 or $\frac{3}{20}$

High Partial Credit
• Sets up equation in k (1^{st} line, or equivalent)
• One error in setting up linear equation in k, and finishes correctly</td></tr>
</table>

P2 Q2	Model Solution – 30 Marks	Marking Notes
(a)	$\frac{1}{\sqrt{n}} = 0·034$ …S1 $\sqrt{n} = \frac{1}{0.034}$ …S2 $n = \left(\frac{1}{0·034}\right)^2 = 865·05 \ldots = 865 \ [\in \mathbb{N}]$ …S3 **OR** $\frac{1}{\sqrt{n}} = 0·034$ …S1 $\frac{1}{n} = (0.034)^2$ …S2 $n = \left(\frac{1}{0·034}\right)^2 = 865·05 \ldots = 865 \ [\in \mathbb{N}]$ …S3	**Scale 10C (0, 3, 7, 10)** Accept a correct value of n, fully verified Note: $n = 841$ to $n = 891$ inclusive will give correct margin of error. Note: can use margin of error between $0·0335$ and $0·0345$. Consider solution as consisting of 3 steps: **Step 1**. Sets $\frac{1}{\sqrt{n}} = 0.034$ **Step 2**. Deals with square root or fraction **Step 3**. Evaluates n *Low Partial Credit* • Work of merit, for example, $0·034$; $\frac{34}{1000}$; or sets $\frac{1}{\sqrt{n}} = 3 \cdot 4$ *High Partial Credit* • 2 steps correct *Note re ** • Apply a * if answer is not rounded to be $\in \mathbb{N}$ (accept rounding up or down in this part)

P2 Q2	Model Solution – 30 Marks	Marking Notes
(b)	**(i)** 95% C.I.: $\bar{x} \pm 1{\cdot}96\frac{s}{\sqrt{n}}$ $= \quad 45{\cdot}4 \pm 1{\cdot}96\left(\frac{3{\cdot}1}{\sqrt{80}}\right)$ $= \quad 45{\cdot}4 \pm 0{\cdot}67 \ldots$ $= \quad 45{\cdot}4 \pm 0{\cdot}7$ [1 D.P.] $= \quad (44{\cdot}7, 46{\cdot}1)$ **(ii)** **Alternative**: "The mean time taken to charge is not 44 minutes" **or** "$\mu \neq 44$" *or equivalent* **Conclusion**: "The mean time taken to charge is not 44 minutes" or: "the company is not correct" or: "reject the null hypothesis" *or equivalent* **Reason**: "44 is outside the 95% C.I." *or equivalent*	**Scale 15D (0, 3, 8, 12, 15)** Accept C.I. even if not given in form suggested. Note: Step 4 and Step 5 can only be considered correct if sufficient work to justify a conclusion is presented in **(i)** or **(ii)** Consider solution as consisting of 5 steps: **Step 1**. $\frac{3{\cdot}1}{\sqrt{80}}$ **Step 2**. 95% C.I. found **Step 3**. Correct alternative hypothesis **Step 4**. Correct conclusion (relative to candidate's work) **Step 5**. Valid reason Note: if only one endpoint of the C.I. is found, Step 2 is not correct. However, if the lower end is found, this is sufficient work to justify a conclusion. Note: if Steps 4 and 5 depend on the test statistic, then (test statistic plus a correct conclusion) is enough for Step 4 to be correct, but test statistic must be compared to $1{\cdot}96$ for Step 5 to be correct. *Low Partial Credit* • Work of merit, for example, mentions 1.96 or $n = 80$ or $\frac{s}{\sqrt{n}}$ or identifies x or s *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 4 steps correct *Note re* * • Apply a * once in **(i)** for incorrect rounding

P2 Q2	Model Solution – 30 Marks	Marking Notes
(c)	We would have more confidence in the conclusion of the second test **or** The conclusion of the second test is more reliable *or any other valid explanation*	**Scale 5A (0, 5)** Accept reference to the appropriate conclusion / test being: • more / less reliable • more / less likely to reject the null hypothesis • more / less confident • more / less accurate or similar

P2 Q3	Model Solution – 30 Marks	Marking Notes
(a)	*Accurate image drawn, with either construction lines (line segments and arcs of circles) and/or relevant lengths measured and marked in.* *Note that, for example, one side could be reflected in l, and the triangle then constructed using the other two side lengths or two angle measurements.* Perpendiculars from vertices of t to l: 4·1 cm, 1·2 cm, 6·9 cm Sides of t: 3·3 cm, 4·8 cm, 6·2 cm Angles in t: 32°, 50°, 98°	**Scale 10C (0, 3, 7, 10)** Tolerance: ± 3 mm *Low Partial Credit* • Work of merit, for example, one relevant line drawn or one relevant measurement indicated • Reasonable image of triangle drawn under a different transformation (translation, rotation, central symmetry, axial symmetry in a line other than the given line) *High Partial Credit* • Correct image (triangle) drawn, but no construction lines or measurements • Two vertices reflected correctly, with the required construction lines / measurements *Note re ** • Apply a * if one construction line / measurement is missing, otherwise correct
(b)	$V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2(h) = 36\,000\pi$ **...S1** so $\frac{1}{3}\pi\frac{h^3}{16} = 36\,000\pi$ **...S2** so $h^3 = 36\,000 \times 48 = 1\,728\,000$ and $h = \sqrt[3]{1\,728\,000} = 120$ **...S3**	**Scale 10D (0, 2, 5, 8, 10)** Using $\pi r^2 h$ or $\frac{4}{3}\pi r^3$ is considered one error. Consider solution as consisting of 3 steps: **Step 1**. Equation in h **Step 2**. Deals with squaring and power of h **Step 3**. Finds h Note: there must be a h^2 or higher power of h in order for Step 3 to be considered correct *Low Partial Credit* • Work of merit, for example, $r = \frac{h}{4}$ • $\frac{1}{3}\pi r^2 h = 36\,000\,\pi$ *Mid Partial Credit* • 1 step correct *High Partial Credit* • 2 steps correct

<table>
<tr><th>P2 Q3</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(c)</td><td>

1. Finding r, Method 1:

[*Letting diagonal of smallest face be x cm and internal diagonal of cuboid be y cm*]

$x^2 = 24^2 + 36^2$ [*Pyth. Thm*]

so $x^2 = 1872$ **…S1**

Then $y^2 = 60^2 + x^2$

so $y = \sqrt{5472} = 12\sqrt{38}$

and $r = \frac{y}{2} = 6\sqrt{38}$ **…S2**

1. Finding r, Method 2:

[*Letting half the diagonal of smallest face be x cm, then r cm is half the internal diagonal of cuboid*]

$x^2 = 12^2 + 18^2$ [*Pyth. Thm*]

so $x^2 = 468$ **…S1**

Then $r^2 = 30^2 + x^2$

so $r = \sqrt{1368}$ **…S2**

$= 6\sqrt{38}$

2. Finding the Volume:

Then $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(6\sqrt{38}\right)^3$ **…S3**

$= \frac{4}{3} \times 6^3 \times 38 \times \sqrt{38}\,\pi$

$= 10\,944\sqrt{38}\,\pi$ [cm^3] **…S4**

</td><td>

Scale 10D (0, 2, 5, 8, 10)

Accept correct answer with no or incorrect units.

Note: If Pythagoras is not used, award *Low Partial Credit* at most.

Note: An answer in the incorrect form (e.g. decimal form) is considered an error.

Consider solution as consisting of 4 steps:

Step 1. Finds length **or** (length)^2 of one relevant diagonal **or** half diagonal

Step 2. Finds r

Step 3. Volume formula fully substituted

Step 4. Volume found, in the correct form

Note: Pythagoras Theorem must be used twice (or $\sqrt{a^2 + b^2 + c^2}$) in order for Step 2 to be considered correct.

Note: Step 4 can only be considered correct if the radius is of the form $a\sqrt{b}$, for constants a and b, and is handled correctly

Low Partial Credit

- Work of merit, for example, Pythagoras Theorem fully substituted, or relevant work on the diagram

Mid Partial Credit

- 2 steps correct

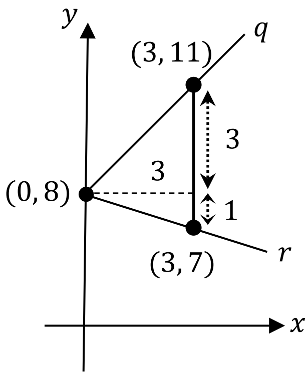
High Partial Credit

- 3 steps correct

*Note re **

- Apply a * if π is omitted, i.e. answer of $10\,944\sqrt{38}$

</td></tr>
</table>

P2 Q4	Model Solution – 30 Marks	Marking Notes
(a)	Answer: No Justification: *Any valid justification, for example:* $m_k = -\frac{3}{2}$ $m_l = \frac{3}{2}$ $-\frac{3}{2} \times \frac{3}{2} \neq -1$	**Scale 10C (0, 3, 7, 10)** **Note:** Conclusion, such as $-\frac{3}{2} \times \frac{3}{2} \neq -1$, is needed for justification to be correct; omission of this is considered an error. *Low Partial Credit* • Correct answer with no supporting work • Work of merit in justification, for example, finds one or two slopes; **or** $m_1 \times m_2 = -1$ *High Partial Credit* • Correct answer **and** work of merit in justification • Work in justification sufficient to support correct answer (including conclusion such as $-\frac{3}{2} \times \frac{3}{2} \neq -1$), **but** no or incorrect answer given.
(b)	*The y-intercept is $(0, 8)$, so critical cases are:*  *Based on the diagram:* $m_q = \frac{3}{3} = 1$ **…S1** $m_r = -\frac{1}{3}$ **…S2** So $m > 1$ or $m < -\frac{1}{3}$ Or ranges are $\left(-\infty, -\frac{1}{3}\right)$ and $(1, \infty)$ **…S3**	**Scale 10D (0, 2, 5, 8, 10)** Accept $m \geq 1$ or $m \leq -\frac{1}{3}$ Consider solution as consisting of 3 steps: **Step 1**. Finds one relevant slope **Step 2**. Finds second relevant slope **Step 3**. Finds inequalities in correct direction (if only 1 slope is found, consider this step correct if the inequality in the correct direction is found) *Low Partial Credit* • Work of merit, for example, diagram drawn with a relevant point plotted; some correct substitution into slope formula or into equation of a line; or finds y-intercept of line *Mid Partial Credit* • 1 step correct *High Partial Credit* • 2 steps correct

<table>
<tr><th>P2 Q4</th><th>Model Solution – 30 Marks</th><th>Marking Notes</th></tr>
<tr><td>(c)</td><td>Constants for perpendicular distance formula:
$a = a,\ b = 1,\ c = -5\ ,\ x_1 = 1,\ y_1 = 0$
$\frac{|a(1)+1(0)-5|}{\sqrt{a^2+1^2}} = 5$ ... S1
so $|a-5| = 5\sqrt{a^2+1}$
so $(a-5)^2 = 25(a^2+1)$
and $a^2 - 10a + 25 = 25(a^2+1)$ ... S2
$a^2 - 10a + 25 = 25a^2 + 25$
$24\,a^2 + 10a = 0$
$a(24a+10) = 0$
$a = 0$ or $24a + 10 = 0$
$a = 0$ or $a = -\frac{5}{12}$ ... S3</td><td>Scale 10D (0, 2, 5, 8, 10)

Accept 0 and $-\frac{10}{24}$

Consider solution as consisting of 3 steps:
Step 1. Equation in a
Step 2. Deals with fraction and squaring
Step 3. Solves for a

Note: if denominator $= 1$, Step 2 is not considered correct (oversimplified)

Note: Step 3 can only be considered correct if there is an a on both sides of the equation once the fraction has been dealt with.

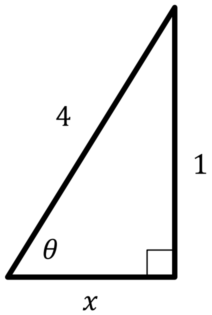
Low Partial Credit
• Work of merit, for example, some correct substitution into perpendicular distance formula

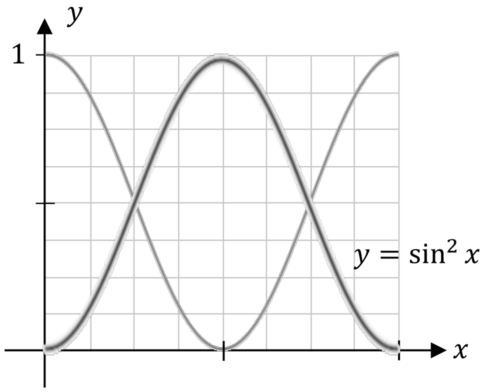
Mid Partial Credit
• 1 step correct

High Partial Credit
• 2 steps correct</td></tr>
</table>

P2 Q5	Model Solution – 30 Marks	Marking Notes
(a)	Centre: $(-2, 5)$ Radius $= \sqrt{2^2 + 5^2 - (-52)} = \sqrt{81} = 9$	**Scale 5C (0, 2, 3, 5)** Accept correct answer without supporting work. *Low Partial Credit:* • Work of merit, for example, identifies g, f, or c; $2g = 4$ • $(2, -5)$ and no further work *High Partial Credit* • Centre or radius correct
(b) (i) (ii)	**(i)** *Any valid combination and the matching circle, for example*: $h = 1, k = 1, r = 2$ $(x-1)^2 + (y-1)^2 = 2^2$ **(ii)** $6 \times 6 \times 6\ \ [= 216]$	**Scale 10D (0, 2, 5, 8, 10)** Accept $\mathbf{6^3}$ or $\mathbf{6 \times 6 \times 6}$ in part **(b)(ii)** Accept correct answers without supporting work. *Low Partial Credit:* • Work of merit, for example, in **(i)**: h, k, or r correct; in **(ii)**, lists 3 or more; indicates 6 *Mid Partial Credit* • 1 part correct • Work of merit in both parts *High Partial Credit* • 1 part correct **and** work of merit in the other part *Note re ** • In **(i)**, apply a * if the equation is a valid equation, but doesn't match the values of h, k, and/or r that the candidate has given

P2 Q5	Model Solution – 30 Marks	Marking Notes
(b) (iii) (iv)	**(iii)** *For these circles, $h = k = r$, for example, $h = k = r = 1$, $h = k = r = 2$, etc.* *Answer* $= 6$ **(iv)** *When $r = 6$, there are no such circles* *When $r = 5$, there is 1 such circle (centre $(6, 6)$)* *When $r = 4$, there are 2^2 such circles* *etc.* *Answer* $= 1^2 + 2^2 + 3^2 + 4^2 + 5^2$ $= 55$	**Scale 15D (0, 3, 8, 12, 15)** Accept $\mathbf{1^2 + 2^2 + 3^2 + 4^2 + 5^2}$ in part **(b)(ii)** *Low Partial Credit:* • Work of merit, for example, in **(iii)** $h = k$, or $h = r$, or $k = r$, centre $(6, 6)$ in **(iv)** correctly identifies the number of circles for a given radius ($1 \leq r \leq 5$); **or** lists any **five** circles with relevant radii and correct corresponding centres; **or** answer for **(ii)** minus answer for **(iii)** *Mid Partial Credit* • **(iii)** correct • Work of merit in both parts *High Partial Credit* • **(iii)** correct **and** work of merit in **(iv)** • **(iv)** correct *Note re* * • Apply a * once if all circles are listed / worked out, but no or incorrect total given, in one or both parts

P2 Q6	Model Solution – 30 Marks	Marking Notes
(a)	**Method 1:** $\sin\theta = \frac{1}{4} = \frac{\text{opp}}{\text{hyp}}$ **… S1**  $x^2 = 4^2 - 1^2 = 15$ so $x = \sqrt{15}$ **… S2** and $\cos\theta = \frac{\sqrt{15}}{4}$ **… S3** **Method 2:** $\cos^2\theta + \sin^2\theta = 1$ So $\cos^2\theta + \left(\frac{1}{4}\right)^2 = 1$ **… S1** Then $\cos^2\theta = 1 - \left(\frac{1}{4}\right)^2$ **… S2** So $\cos\theta = \sqrt{\frac{15}{16}}$ or $\frac{\sqrt{15}}{4}$ **… S3**	**Scale 10C (0, 3, 7, 10)** Clear calculator work is not considered correct work / work of merit Consider **Method 1** of the solution as consisting of 3 steps: **Step 1**. Identifies hypotenuse **Step 2**. Finds length of third side **Step 3**. Finds $cos\ \theta$ Note: Step 3 can only be considered correct if Pythagoras Theorem is used in Step 2. Consider **Method 2** of the solution as consisting of 3 steps: **Step 1**. Subs into $cos^2\ \theta + sin^2\ \theta = 1$ **Step 2**. Isolates $cos^2\ \theta$ **Step 3**. Finds $cos\ \theta$ *Low Partial Credit:* • Work of merit, for example, right-angled triangle drawn; some correct substitution into Pythagoras' theorem *High Partial Credit* • 2 steps correct

P2 Q6	Model Solution – 30 Marks	Marking Notes
(b)		**Scale 5C (0, 2, 3, 5)** Consider solution as requiring 3 aspects: **Asp 1**. correct start, middle, and end points **Asp 2**. local max at middle point **Asp 3**. correct concavity (up to $\frac{1}{4}$ of domain; from $\frac{1}{4}$ to $\frac{3}{4}$; and from $\frac{3}{4}$ to the end) *Low Partial Credit:* • Work of merit, for example, $\sin^2 x = 1 - \cos^2 x$, or 1 point worked out or correctly plotted *High Partial Credit* • 2 aspects correct • Graph of $y = -\cos^2 x$ drawn *Note re ** • Apply a * if more than one graph is drawn and the correct graph is not labelled

P2 Q6	Model Solution – 30 Marks	Marking Notes
(c)	*Let P and Q be subtended by angles A and B, respectively. Then*: $P=(\cos A,\sin A)$; $Q=(\cos B,\sin B)$. Then $\|PQ\|^2$ is: $(\cos A-\cos B)^2+(\sin A-\sin B)^2$ **...S1** $=\cos^2 A-2\cos A\cos B+\cos^2 B$ $+\sin^2 A-2\sin A\sin B+\sin^2 B$ $=2-2\cos A\cos B-2\sin A\sin B$ **...S3** [as $\cos^2\theta+\sin^2\theta=1$] By the cosine rule: $\|PQ\|^2=1^2+1^2-2(1)(1)\cos(A-B)$ **...S2** $=2-2\cos(A-B)$ Equating these: $2-2\cos(A-B)$ $=2-2\cos A\cos B-2\sin A\sin B$ So $\cos(A-B)=\cos A\cos B+\sin A\sin B$ **...S4** **OR** *Instead of cosine rule:* Rotate $[PQ]$ about 0 until Q coincides with $(1,0)$. Let the image of P be P'. Then the square of the distance P' to $(1,0)$ is: $(\cos(A-B)-1)^2+(\sin(A-B)-0)^2$ **...S2** $=\cos^2(A-B)-2\cos(A-B)+1$ $+\sin^2(A-B)$ $=2-2\cos(A-B)$ *Then equate as above.*	**Scale 15D (0, 3, 8, 12, 15)** Accept a correct proof, even if inconsistent with labelling on diagram Consider solution as consisting of 4 steps: **Step 1**. Finds one (unsimplified) expression for $\|PQ\|^2$ **Step 2**. Finds a second (unsimplified) expression for $\|PQ\|^2$ **Step 3**. Simplifies one expression for $\|PQ\|^2$ **Step 4**. Finishes proof *Low Partial Credit* • Work of merit, for example, correctly labels one the angles A, B or A-B; or radius of 1 written on line from centre to P or from centre to Q *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct

P2 Q7	Model Solution – 50 Marks	Marking Notes
(a) (i)	Area = $\frac{1}{2}(a)(b)\sin C$ a = 12, b = 20, C = 58° $A \quad = \frac{1}{2}(12)(20)\sin 58°$ **... HPC** $= 101{\cdot}7\,...$ $= 102$ [cm²] [$\in \mathbb{N}$]	**Scale 5C (0, 2, 3, 5)** Accept correct answer with incorrect or no unit *Low Partial Credit* • Work of merit, for example, some correct substitution into area formula *High Partial Credit* • Full substituted area formula *Note re ** • Apply a * for incorrect rounding
(a) (ii)	$x^2 = 12^2 + 20^2 - 2(12)(20)\cos 58°$ **...HPC** $= 289{\cdot}6\,...$ $x \quad = \sqrt{289{\cdot}6\,...}$ $= 17{\cdot}01\,...$ **... F*** $= 17$ [cm] [$\in \mathbb{N}$]	**Scale 10C (0, 3, 7, 10)** Note that a unit is not required in the answer, as $x \in \mathbb{N}$ Accept 17·0 *Low Partial Credit* • Work of merit, for example, some correct substitution into Cosine formula *High Partial Credit* • Fully substituted Cosine formula *Note re ** • Apply a * for incorrect rounding

P2 Q7	Model Solution – 50 Marks	Marking Notes
(b)	$180 - (34 + 59) = 180 - 93 = 87°$ **... S1** $\dfrac{y}{\sin 59} = \dfrac{72}{\sin 87}$ **... S2** so $y = \frac{72 \sin 59}{\sin 87}$ **... S3** $= 61{\cdot}8 \ldots$ **... F*** $= 62 \; [\in \mathbb{N}]$ **... S4**	**Scale 10D (0, 2, 5, 8, 10)** Note that a unit is not required in the answer, as $y \in \mathbb{N}$ Consider solution as consisting of 4 steps: **Step 1**. Finds third angle in triangle **Step 2**. Fully substituted Sine Rule **Step 3.** Isolates y **Step 4**. Evaluates y *Low Partial Credit* • Work of merit, for example, mentions 180°; some correct substitution into Sine rule *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Note re* * • Apply a * for incorrect rounding

P2 Q7	Model Solution – 50 Marks	Marking Notes
(c) (i)	**Method 1:** *By Pythagoras Theorem:* $(n+10)^2 = (n+5)^2 + n^2$ **… S1** so $n^2 + 20n + 100 = n^2 + 10n + 25 + n^2$ and $n^2 - 10n - 75 = 0$ **… S2** so $(n-15)(n+5) = 0$ **… S3** i.e. $n = 15$ [as $n > 0$] **… S4** **Method 2:** Triangle must be similar to 3, 4, 5, i.e. sides are $3x$, $4x$, $5x$ Therefore $4x - 3x = (n+5) - n$ So $x = 5$, and $n = 3x = 15$	**Scale 15D (0, 3, 8, 12, 15)** Note that a unit is not required in the answer, as $n \in \mathbb{N}$ Consider solution as consisting of 4 steps: **Step 1**. Sets up Pythagoras Theorem **Step 2**. Equation in the form $an^2 + bn + c = 0$ **Step 3**. Factorised quadratic / fully substituted quadratic formula **Step 4**. Finds n Note: a, b, and c all need to be non-zero in Step 2 in order for Steps 3 and/or 4 to be considered correct *Low Partial Credit* • Work of merit, for example, trials one value of n; or writes down 3, 4, 5; or some substitution into Pythagoras; some correct squaring of n+5 *Mid Partial Credit* • 2 steps correct • $3x$, $4x$, $5x$ (Method 2) *High Partial Credit* • 3 steps correct • $4x - 3x = (n+5) - n$, or similar *Note re ** • Apply a * if 15 and -5 are found, and 15 is not selected
(c) (ii)	Answer: $60°$ Justification: *Any valid justification, for example:* As n gets very big, the triangle gets closer and closer to being equilateral.	**Scale 5B (0, 2, 5)** *Partial Credit* • Answer correct, but insufficient justification • Incorrect answer, but indicates equilateral triangle

<table>
<tr><th>P2 Q7</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(d)</td><td>
Triangle similar to 12, 24, 30 has sides in same ratio.

Smallest such triangle is $2, 4, 5$

Based on this smallest triangle, all the similar triangles are,:

$2, 4, 5$
$\times 2$: $4, 8, 10$
$\times 3$: $6, 12, 15$
$\dots$
$\times 20$: $40, 80, 100$

There are 20 triangles here, but we need to exclude $12, 24, 30$, so

Answer $= 19$
</td><td>
Scale 5D (0, 2, 3, 4, 5)
Consider solution as consisting of 4 steps:

Step 1. Finds one non-congruent similar triangle
Step 2. Finds smallest non-congruent similar triangle
Step 3. Finds / indicates biggest non-congruent similar triangle, based on their (smaller) similar triangle
Step 4. Finds n, based on all scalings of their smallest triangle.

Note: If the smallest similar triangle presented is bigger than or equal to the triangle with sides of length 12, 24, and 30, none of Steps 2, 3, or 4 are considered correct.

Low Partial Credit
• Work of merit, for example, states ratio of sides is 12:24:30

Mid Partial Credit
• 2 steps correct
• presents five non-congruent similar triangles

High Partial Credit
• 3 steps correct

Note re *
• Apply a * if the given triangle is not excluded (i.e. answer $= 20$)
</td></tr>
</table>

P2 Q8	Model Solution – 50 Marks	Marking Notes
(a)	Answer: $0 \cdot 8$ Justification: *valid justification; should refer to* *(1) direction: values increasing* ***and*** *(2) strength: moderate/fairly spread out* *(or acceptable equivalent);* ***or*** *specifically exclude the other listed values of* r	**Scale 5C (0, 2, 3, 5)** Accept “positive” instead of “increasing” Accept broad range of descriptions for the strength (from “weak” to “moderate” to “strong”) *Low Partial Credit* • $0 \cdot 25$ or $0 \cdot 95$ selected • Work of merit in justification, for example, mentions positive or moderate / weak / strong *High Partial Credit* • Answer of $0 \cdot 8$ • Direction **and** strength acceptable in justification • $0 \cdot 25$ or $0 \cdot 95$ selected **and** strength acceptable in justification *Note re ** • Apply a * for an answer of $0 \cdot 8$ **and** strength acceptable in justification (but omits increasing/positive)

P2 Q8	Model Solution – 50 Marks	Marking Notes
(b)(i) **(b)(ii)** **(b)(iii)**	**(b)(i), (ii)** *Point $P(13, 10{\cdot}7)$ plotted and labelled, and reasonable line of best fit drawn through P.* **(b)(iii)** *Reasonable value read from their own line. Likely around 6.*	**Scale 5C (0, 2, 3, 5)** Tolerance for **(iii)**: answer should be within the horizontal gridlines for the Energy value. In **(iii)**, accept answer as a range of values, as long as full range given is within tolerance Consider solution as consisting of 3 steps: **Step 1**. Point P plotted and labelled **Step 2**. Reasonable line of best fit **Step 3**. Value read from line of best fit. *Low Partial Credit* • Work of merit, for example, relevant vertical/horizontal line drawn on the diagram *High Partial Credit* • 2 steps correct *Note re ** • Apply a * if point P is not labelled • Apply a * if the line of best fit does not go through the point P • Apply a * if work is not shown on the diagram for **(iii)**

P2 Q8	Model Solution – 50 Marks	Marking Notes
(b)(iv)	*Equation found using* $(13, 10\cdot7)$ *and their answer to part* ***(b)(iii)*** *For example, if answer to* ***(b)(iii)*** *is* $(11, 6)$*:* **Method 1** Slope $= \frac{10\cdot7-6}{13-11} = \frac{4\cdot7}{2} = 2\cdot35$ **... S1** Equation: $y - 6 = 2\cdot35(x - 11)$ **... S2** $y = 2\cdot35x - 19.85$ **... S3** **Method 2** $(13, 10.7)\ and\ (11,6)\ \in\ y = mx + c$ $10.7 = m(13) + c$ $6 = m(11) + c$ **... S1** Therefore, $2\,m = 4.7$ $m = \frac{4\cdot7}{2} = 2\cdot35$ **... S2** $c = -19.85$ Equation: $y = 2\cdot35x - 19.85$ **...S3**	**Scale 10C (0, 3, 7, 10)** Consider solution as consisting of 3 steps: **Method 1** **Step 1**. Finds slope of line **Step 2**. Fully substituted formula for equation of a line (no unknown constants) **Step 3**. Equation in the required form **Method 2** **Step 1.** Writes 2 equations in m and c **Step 2.** Finds value of m **or** c **Step 3.** Equation in the required form *Low Partial Credit* • Work of merit, for example, some correct substitution into slope formula or equation of line formula; brings down point from **b(i)** or **b(iii)** *High Partial Credit* • 2 steps correct *Note re ** • Apply a * once if the answer to **(b)(iii)** and/or the value of P are not used.

P2 Q8	Model Solution – 50 Marks	Marking Notes
(c)	**Method 1**: $P(\text{at least 1}) = 1 - P(\text{none in common})$ $= 1 - \frac{126C2}{128C2}$ …… **HP** $= 1 - \frac{126\times125}{128\times127}$ ……. **HP** $= 1 - \frac{7875}{8128} = \frac{253}{8128}$ $= 0{\cdot}0311\ldots = 0{\cdot}031$ [3 D.P.] **Method 2**: $P(\text{at least 1}) = \frac{127+127-1}{128C2}$ …….**HP** $= \frac{253}{\left(\frac{128\times127}{2}\right)} = \ldots = 0{\cdot}031$ [3 D.P.] **Method 3**: $P(\text{at least 1}) = \frac{2\times127+126\times2}{128\times127}$ …… **HP** $= \frac{506}{16256} = \ldots = 0{\cdot}031$ [3 D.P.] **Method 4**: $P(\text{at least 1}) = \frac{2C1\times126C1 + 2C2\times126C0}{128C2}$ …… **HP** $= \frac{253}{8128} = \ldots = 0{\cdot}031$ [3 D.P.] **Method 5**: $P(\text{at least 1}) = \frac{2}{128}\times\frac{1}{127} + \frac{2}{128}\times\frac{126}{127}\times 2$ …**HP** $= \frac{1}{8128} + \frac{63}{2032} = \ldots = 0{\cdot}031$ [3 D.P.]	**Scale 10C (0, 3, 7, 10)** *Low Partial Credit* • Work of merit, for example, $128C2, 126C2$; one relevant fraction, for example $\frac{126}{128}, \frac{125}{128}$; a relevant calculation, for example 128×127; states $1 - P(none)$ *High Partial Credit* • Correct answer, not evaluated • P(none in common) = $\frac{7875}{8128}$ • One error, finished correctly, for example, evaluates $1 - \frac{126\times125}{128\times128}$ or $\frac{127+126}{128\times127}$ *Note re ** • Apply a * for incorrect rounding

<table>
<tr><th>P2 Q8</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(d)(i)</td><td>$t = 7$

$H(7) = 4{\cdot}5\sin\left(\frac{14\pi}{365}\right) + 12$

$= 0{\cdot}540\ldots + 12$

$= 12{\cdot}54$ [2 D.P.]</td><td>Scale 5C (0, 2, 3, 5)
Consider solution as consisting of 3 steps:

Step 1. Finds $t = 7$
Step 2. Substitutes $H(7)$
Step 3. Evaluates $H(7)$

Note: if the calculator is in a mode other than radian mode, this is considered an error, so Step 3 will not be correct.

Low Partial Credit
• 1 step correct

High Partial Credit
• 2 steps correct
• Evaluates $H(k)$ correctly, for $0 < k < 365$, $k \neq 7$, $k \in \mathbb{N}$

Note re *
• Apply a * for incorrect rounding</td></tr>
</table>

P2 Q8	Model Solution – 50 Marks	Marking Notes
(d)(ii)	Letting $A = \frac{2\pi t}{365}$: $4{\cdot}5 \sin A + 12 = 8$ so $\sin A = -\frac{4}{4{\cdot}5} = -\frac{8}{9}$ .... **S1** so $\text{Ref}(A) = \sin^{-1}\left(\frac{8}{9}\right) = 1{\cdot}0949\ldots$ [radians] .... **S2** **Method 1 to finish**: $A = \pi + 1{\cdot}0949\ldots$ So $\frac{2\pi t}{365} = \pi + 1{\cdot}0949\ldots$ i.e. $t = \frac{365(\pi+1{\cdot}0949\ldots)}{2\pi} = 246{\cdot}10\ldots$ .... **S3** **Or** $A = 2\pi - 1{\cdot}0949$ ... **S4** $\frac{2\pi t}{365} = 2\pi - 1{\cdot}0949\ldots$ i.e. $t = \frac{365(2\pi-1{\cdot}0949\ldots)}{2\pi} = 301{\cdot}39\ldots$ So first day: $t = 247$ [first day after 246·10] and last day: $t = 301$ [last day before 301·39] ..... **S5** **Method 2 to finish**: $x = \frac{365}{2\pi} \times 1{\cdot}0949\ldots = 63{\cdot}60\ldots$ ..... **S3** Then $t = \frac{365}{2} + 63{\cdot}60\ldots = 182{\cdot}5 + 63{\cdot}60\ldots$ $= 246{\cdot}10\ldots$ ..... **S4** **Or** $t = 365 - 63{\cdot}60\ldots = 301{\cdot}39\ldots$ So first day: $t = 247$ [first day after 246·10] and last day: $t = 301$ [last day before 301·39] **S5**	**Scale 15D (0, 3, 8, 12, 15)** For **Method 1**, consider solution as consisting of 5 steps: **Step 1**. Finds $sin\,A = -\frac{8}{9}$ **Step 2**. Finds reference value of A **Step 3**. Finds first value of t **Step 4**. Finds second value of A **Step 5**. Finds second value of t For **Method 2**, consider solution as consisting of 5 steps: **Step 1**. Finds $sin\,A = -\frac{8}{9}$ **Step 2**. Finds reference value of A **Step 3**. Finds $x = 63{\cdot}60\ldots$ **Step 4**. Finds first value of t **Step 5**. Finds second value of t Note: for either method, if the calculator is in a mode other than radian mode, this is considered an error, so Step 2 will not be correct, unless π is replaced by 180 throughout *Low Partial Credit* • Work of merit, for example, sets equation = 8 *Mid Partial Credit* • 2 steps correct • 1 correct answer with no supporting work *High Partial Credit* • 3 steps correct • 2 correct answers with no supporting work • Evaluates $H(246)$ and $H(247)$, and selects 247 • Evaluates $H(301)$ and $H(302)$, and selects 301 *Note re ** • Apply a * once for incorrect rounding

<table>
<tr><th>P2 Q9</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

A valid set of statements and reasons, for example:

<table>
<tr><th></th><th>Statement</th><th>Reason</th></tr>
<tr><td>S1</td><td>$|PA| = |PB|$</td><td>Radii [of k]</td></tr>
<tr><td>S2</td><td>$|PO| = |PO|$</td><td>Common side</td></tr>
<tr><td>S3</td><td>$|\angle A| = |\angle B| = 90°$</td><td>Tangents are perpendicular to radii at point of contact</td></tr>
<tr><td>C</td><td>Therefore, $\Delta\ POA$ and $\Delta\ POB$ are congruent</td><td>RHS
OR
SSS, as $|AO| = |BO|$ by Pythagoras Theorem</td></tr>
</table>

</td><td>

Scale 10D (0, 2, 5, 8, 10)

Consider solution as consisting of 6 parts: 2 statements and 4 reasons

Note: For each statement on the left, must give matching reason on the right.

Work indicated on the diagram is generally taken as a statement without a reason

Low Partial Credit

- Work of merit, for example, relevant work on the diagram

Mid Partial Credit

- 2 parts correct

High Partial Credit

- 4 parts correct

</td></tr>
<tr><td>(b) (i)</td><td>

Method 1:

$\tan 70° = \dfrac{|AO|}{12}$ **...S1**

so $|AO| = 12\tan 70°$ **...S2**

$= 32{\cdot}96\ldots$ **...* ***

$= 33{\cdot}0$ cm [1 D.P.] **...S3**

Method 2:

$\dfrac{|AO|}{\sin 70°} = \dfrac{12}{\sin 20°}$ **...S1**

so $|AO| = \dfrac{12\sin 70°}{\sin 20°}$ **...S2**

$= 32{\cdot}96\ldots$ **...* ***

$= 33{\cdot}0$ cm [1 D.P.] **...S3**

</td><td>

Scale 5C (0, 2, 3, 5)

Accept 33 (i.e. 33·0 not required)

Consider solution as consisting of 3 steps:

Step 1. Sets up tan ratio / Sine rule fully substituted

Step 2. Isolates $|AO|$

Step 3. Evaluates $|AO|$

Note: if the calculator is in a mode other than degree mode, this is considered an error.

Low Partial Credit

- Work of merit, for example, states adj = 12; **or** shows 70° on the diagram

High Partial Credit

- 2 steps correct

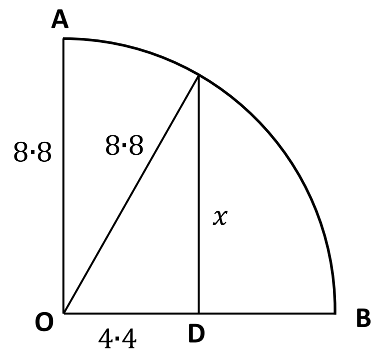
*Note re **

- Apply a * once for incorrect rounding
- Apply a * for incorrect or no unit

</td></tr>
</table>

P2 Q9	Model Solution – 50 Marks	Marking Notes
(b) (ii)	$\lvert AO\rvert = 33$ cm so $\lvert OD\rvert = \frac{8}{12} \times 33 = 22$ cm **.... S1** $\lvert AB\rvert_{\text{arc}} = \frac{220}{360} \times 2\pi(12)$ **.... S2** $= \frac{44}{3}\pi$ [cm] $\lvert CD\rvert_{\text{arc}} = \frac{220}{360} \times 2\pi(8)$ **.... S3** $= \frac{88}{9}\pi$ [cm] $\lvert AD\rvert + \lvert BC\rvert = 2(33 + 22) = 110$ [cm] So total length $= 110 + \frac{44}{3}\pi + \frac{88}{9}\pi$ $= 186{\cdot}7 \ldots$ $= 187$ [cm] [$\in \mathbb{N}$] **.... S4** **Method 2** $\lvert AB\rvert_{\text{arc}} = \frac{220}{360} \times 2\pi(12)$ **.... S2** $= \frac{44}{3}\pi$ [cm] $\lvert AB\rvert_{arc} + \lvert AO\rvert + \lvert OB\rvert = \frac{44}{3}\pi + 33 + 33 = 112{\cdot}07\ldots$ $\lvert CD\rvert_{arc} + \lvert OD\rvert + \lvert OC\rvert = 112.07 \times \frac{8}{12} = 74.717$ **...S1 & S3** So total length $= 186{\cdot}7 \ldots = 187$ [cm] [$\in \mathbb{N}$] **...S4** **Method 3** $\lvert AB\rvert_{\text{arc}} = \frac{220}{360} \times 2\pi(12)$ **...S2** $= \frac{44}{3}\pi$ [cm] $\lvert AB\rvert_{arc} + \lvert AO\rvert + \lvert OB\rvert = \frac{44}{3}\pi + 33 + 33 = 112.07..$ Total length $= 112.07 \times 1\frac{8}{12} = 186.7.. = 187$ [cm] **...S1, 3 & 4**	**Scale 15D (0, 3, 8, 12, 15)** Accept correct answer with incorrect or no unit Consider solution as consisting of 4 steps: **Step 1**. Finds $\lvert OD\rvert$ or $\lvert OC\rvert$ [unsimplified] **Step 2**. Finds $\lvert AB\rvert_{arc}$ [unsimplified] **Step 3**. Finds $\lvert CD\rvert_{arc}$ [unsimplified] **Step 4**. Finds total length Accept answer if calculator is in mode other than degree mode, if the same error was penalised in part **(b)(i)**, as answer can be got from scaling answer from part **(b)(i)** *Low Partial Credit* • Work of merit, for example, length of $[OB]$ marked on the diagram; mentions $\frac{8}{12}$ *Mid Partial Credit* • 2 steps correct • Finds $\lvert AD\rvert$ or $\lvert BC\rvert$ *High Partial Credit* • 3 steps correct • $\lvert AD\rvert + \lvert BC\rvert + \lvert AB\rvert_{\text{arc}}$ evaluated • $\lvert AD\rvert + \lvert BC\rvert + \lvert CD\rvert_{\text{arc}}$ evaluated *Note re ** • Apply a * once for incorrect rounding

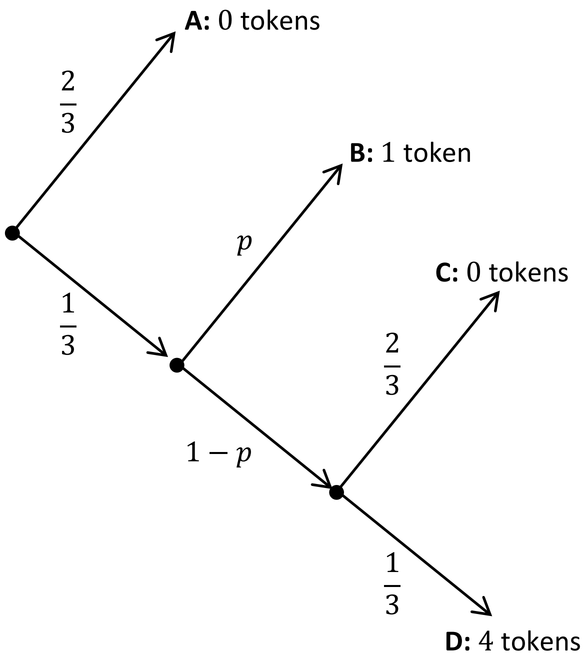
P2 Q9	Model Solution – 50 Marks	Marking Notes
(c)	$j = 55 - 2x$ **...S1** $k = 60 - x$ **...S2** So $V = x(60 - x)(55 - 2x)$ **...S3** $= x(3300 - 120x - 55x + 2x^2)$ **...S4** $= 2x^3 - 175x^2 + 3300x$ **...S5**	**Scale 10D (0, 2, 5, 8, 10)** Consider solution as consisting of 5 steps: **Step 1**. Identifies j **Step 2**. Identifies k **Step 3**. Substitutes into V **Step 4**. Multiplies two linear factors (simplification not required) **Step 5**. V in correct form Note: For Steps 3, 4, and 5, all lengths being used must be in terms of x only in order for these to be considered correct. *Low Partial Credit* • Work of merit, for example, relevant work on the diagram; $x + j + x = 55$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 4 steps correct

P2 Q9	Model Solution – 50 Marks	Marking Notes
(d)	**Method 1**: *[Ignore all area left of **A**, as it will be correctly found when the Trapezoidal Rule is used.]* Actual area $= \frac{1}{4}\pi(8{\cdot}8^2)$ $= 60{\cdot}82\ldots$ [m^2] **...S1** $\lvert$**OD**$\rvert = \frac{8{\cdot}8}{2} = 4{\cdot}4$ **...S2** Estimated area:  $x = \sqrt{8{\cdot}8^2 - 4{\cdot}4^2} = 7{\cdot}621\ldots$ So estimated area: $= \frac{4{\cdot}4}{2} \times [8{\cdot}8 + 0 + 2(7{\cdot}621\ldots)]$ $= 52{\cdot}8924\ldots$ **...S3** I.e. error $= 60{\cdot}82\ldots - 52{\cdot}89..$ $= 8$ [m^2] [$\in \mathbb{N}$] **...S4** **Method 2**: calculate entire area (**S1** & **S2** as above) *Area to the left of **A** is:* $\frac{4{\cdot}4}{2} \times [6{\cdot}7 + 8{\cdot}8 + 2(9{\cdot}1 + 10{\cdot}2 + 8{\cdot}8)]$ $= 157{\cdot}74$ [m^2] So actual total area $= 157{\cdot}74 + 60{\cdot}82\ldots$ $= 218{\cdot}56\ldots$ [m^2] and estimated total area by trapezoidal rule $= 210{\cdot}63\ldots$ [m^2] **... S3** I.e. error $= 8$ [m^2] **...S4**	**Scale 10D (0, 2, 5, 8, 10)** Consider solution as consisting of 4 steps: **Step 1**. Finds area of quarter circle **Step 2**. Finds width of strip ($4{\cdot}4$) **Step 3**. Finds what estimated area of quarter circle, or of whole region, would be, if trapezoidal rule were used **Step 4**. Finds error Step 4 cannot be considered correct unless Step 1 and Step 3 are both completed (with or without errors) *Low Partial Credit* • Work of merit, for example, some correct substitution into trapezoidal rule/area of a circle formula; relevant work on the diagram *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Note re ** • Apply a * for incorrect rounding

P2 Q10	Model Solution – 50 Marks	Marking Notes
(a) (i)	$x = 50,\ \mu = 46.02,\ \sigma = 11.72$ **… S1** $z = \dfrac{50 - 46 \cdot 02}{11 \cdot 72}$ $= 0{\cdot}3395 \ldots = 0{\cdot}34$ or $0{\cdot}33$ **… S2** $P(Z < 0{\cdot}34) = 0.6331$ **… S3** So $P(X > 50) = P(Z > 0{\cdot}34)$ $= 1 - P(Z < 0{\cdot}34)$ $= 1 - 0{\cdot}6331$ $= 0{\cdot}3669$ **… F*** $= 37$ [%] $[\in \mathbb{N}]$ **… S4** **Or** $P(Z < 0{\cdot}33) = 0.6293$ **… S3** So $P(X > 50) = P(Z > 0{\cdot}33)$ $= 1 - P(Z < 0{\cdot}33)$ $= 1 - 0{\cdot}6293$ $= 0{\cdot}3707$ **… F*** $= 37$ [%] $[\in \mathbb{N}]$ **… S4**	**Scale 10D (0, 2, 5, 8, 10)** Consider solution as consisting of 4 steps: **Step 1**. Identifies x, μ, and σ **Step 2**. Finds z-score ($0{\cdot}33$ or $0{\cdot}34$) **Step 3**. Finds $P(z < a)$ **Step 4**. Finds required probability *Low Partial Credit* • Work of merit, for example, normal curve drawn with some relevant work, or some correct substitution into $z = \frac{x-\mu}{\sigma}$ • Identifies μ or σ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Note re ** • Apply a * once for incorrect or no rounding, and/or for answer as a decimal rather than a percentage

P2 Q10	Model Solution – 50 Marks	Marking Notes
(a) (ii)	$P(Z < k) = 0{\cdot}9$, so $k = 1{\cdot}28$ **or** $1{\cdot}29$ **... S1** $[k =]\ \frac{X-46{\cdot}02}{11{\cdot}72}$ **... S2** so $\frac{X-46{\cdot}02}{11{\cdot}72} = 1{\cdot}28$ **... S3** $X - 46{\cdot}02 = 1{\cdot}28 \times 11{\cdot}72$ $X = 46{\cdot}02 + 1{\cdot}28 \times 11{\cdot}72$ $= 61{\cdot}0216$ **... F*** $= [€]\ 61\ [\in \mathbb{N}]$ **... S4** **OR** $\frac{X-46{\cdot}02}{11{\cdot}72} = 1{\cdot}29$ **... S3** $X = 46{\cdot}02 + 1{\cdot}29 \times 11{\cdot}72$ $= 61{\cdot}1388$ **... F*** $= [€]\ 61\ [\in \mathbb{N}]$ **... S4**	**Scale 10D (0, 2, 5, 8, 10)** Consider solution as consisting of 4 steps: **Step 1**. Finds z-score ($1{\cdot}28$ or $1{\cdot}29$) **Step 2**. Identifies $x = X$, μ, and σ **Step 3**. Equation in X **Step 4**. Solves for X *Low Partial Credit* • Work of merit, for example, normal curve drawn with some relevant work, or some correct substitution into $z = \frac{x-\mu}{\sigma}$ • Identifies μ or σ [if not already awarded in part **(a)(i)**] *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct *Note re ** • Apply a * for incorrect or no rounding
(b) (i) (ii)	**(i)** $\frac{2}{6}$ **or** $\frac{1}{3}$ **(ii)** $\left(\frac{2}{3}\right)^4 \times \left(\frac{1}{3}\right) = \frac{16}{243}$	**Scale 5C (0, 2, 3, 5)** *Low Partial Credit* • Work of merit, for example, in **(i)**: states 2 favourable outcomes, or finds probability of **not** getting a token on one go; in **(ii)**: multiplies two relevant fractions, or finds $P(\text{gets nothing})$ *High Partial Credit* • **(i)** correct **and** work of merit in **(ii)** • **(ii)** correct *Note re ** • Apply a * if probability is as a decimal rather than a fraction in part **(ii)**

P2 Q10	Model Solution – 50 Marks	Marking Notes
(b) (iii)	$P(4 \text{ or more}) \quad = P(4 \text{ or } 5)$ $= P(4) + P(5)$ $= \left[5C4 \times \left(\frac{1}{3}\right)^4 \times \left(\frac{2}{3}\right)^1\right] + \left(\frac{1}{3}\right)^5$ $= \frac{10}{243} + \frac{1}{243}$ $= \frac{11}{243}$	**Scale 10D (0, 2, 5, 8, 10)** *Low Partial Credit* • Work of merit, for example, $P(4) + P(5)$, or $5C4$, or $5C5$ *Mid Partial Credit* • $P(4)$ correctly substituted • $P(5)$ fully correct $\left(\frac{1}{243}\right)$ • Two of $P(0)$, $P(1)$, $P(2)$, and $P(3)$ correctly evaluated *High Partial Credit* • $P(4)$ fully correct….$\frac{10}{243}$ • $P(5)$ correct **and** one aspect of $P(4)$ correct (binomial coefficient, power of $\frac{1}{3}$, power of $\frac{2}{3}$) • $P(\text{less than } 4)$ evaluated *Note re ** • Apply a * if probability is as a decimal rather than a fraction

P2 Q10	Model Solution – 50 Marks	Marking Notes
(c) (i)	 $P(1 \text{ token}) \quad = \left[\frac{1}{3} \times p \times 1\right]$ **...S2 (& S1)** $4 \times P(4 \text{ tokens}) = \left[\frac{1}{3} \times (1-p) \times \frac{1}{3} \times 4\right]$ **...S3** $\mathbb{E}[\text{playing}] \quad = \frac{p}{3} + \frac{4(1-p)}{9}$ $= \frac{3p+4-4p}{9}$ $= \frac{4-p}{9}$	**Scale 10D (0, 2, 5, 8, 10)** Consider solution as consisting of 4 steps: **Step 1**. Tree diagram (this can be assumed done if 1 other step is completed) **Step 2**. Finds $P(1\ token)$ **Step 3**. Finds $4 \times P(4\ tokens)$ using $1-p$ **Step 4**. Answer (in correct form) Note: if $1-p$ is not used in Step 3, then Step 4 cannot be considered correct. *Low Partial Credit* • Work of merit, for example, some work of merit in tree diagram, **or** states $x.p(x)$ • Probability of rolling again = $1-p$ *Mid Partial Credit* • 2 steps correct *High Partial Credit* • 3 steps correct
(c) (ii)	Player should roll again [if they get a 5 or a 6]	**Scale 5A (0, 5)**